

# HEAT TRANSFER

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**Third Edition**

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*To Brigid*  
*For your patience and understanding.*

*To Kaori*  
*For your loving support.*



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# PREFACE

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For the third edition of *Heat Transfer* Anthony Mills is joined by Carlos Coimbra as a co-author. Professor Coimbra brings to this venture the perspective and skills of a younger generation of heat transfer educators and his own special expertise in areas of heat transfer research. The second editions of the texts by Anthony Mills included *Heat Transfer*, *Basic Heat and Mass Transfer* and *Mass Transfer*, in order to provide texts suitable for a variety of courses and instructor's needs. In bringing out a third edition of this material we are added the new title *Basic Heat Transfer* in response to requests for a text suitable for a junior or senior level course that covers heat transfer only. This goal was accomplished by essentially removing the chapter on mass transfer from *Basic Heat and Mass Transfer*, which reduced the text length from 1001 pages to 828 pages. We have designated *Basic Heat Transfer* as a third edition to avoid possible confusion since it is based on the third edition updated material of the textbook series. The third edition of *Heat Transfer* can be used for both introductory and more advanced (including graduate) courses.

Sixteen years after the second edition was published, a new edition of these materials is perhaps overdue, but in a mature field such as heat transfer, it is not at all clear what topics should be introduced, and then what topics should be removed to retain an acceptable length for an introductory text. As a result, our main motivation in publishing a third edition has been a different consideration.

Our concern was the excessive prices of college textbooks, which in recent years have destroyed the established role played by these texts in the education of engineering students. Traditionally, students bought a required textbook, became familiar with it in taking the course, and then retained the book as a tool for subsequent courses and an engineering career. Nowadays the pattern is for a student to sell the textbooks back to the university bookstore at the end of the course in order to obtain funds for buying textbooks for the next term. Alternatively, electronic versions of portions of the text are used during the course, or course readers containing selected material from the text may be used. It is particularly frustrating to instructors of subsequent design and laboratory courses to find that the students no longer have appropriate textbooks. Also, the traditional role formerly played by textbooks as professional manuals for engineering practice has been significantly affected. Basic

methodology and data are more easily and reliably obtained from a familiar text than from an internet search.

In an attempt to mitigate these problems and improve the experience of our engineering students we decided to retain creative and publishing rights over the content of this book for this and future editions. A company called Temporal Publishing LLC was created to publish quality engineering textbooks at more reasonable prices.<sup>1</sup> This entailed first converting the previous edition to LaTeX, which we could then modify efficiently. Since the conversion proved to be a major project in itself, our objective with this third edition is rather modest. We have focused on corrections, clarifications, minor updates and the production of a dedicated companion website.<sup>2</sup> We envisage this website to be an integral part of the project and hope to make it a really useful adjunct to the text, for both students and instructors. The website contains links to the dedicated software HT that automates most of the calculations done in the text, instructor aides (such as complete solutions manual for adoptees of the text, additional examples and exercises, presentations, etc.) and a compilation of answers to odd-numbered exercises to assist self-study by students. We will be continuously adding new technical content to the website while we work on future editions of the textbook. Also, given our closer association with the print-on-demand process, it will be easy for the authors to implement small improvements in subsequent printings of this edition. We certainly welcome input and suggestions from users to improve our product.

In preparing this new edition we have had valuable assistance from:

Marius Andronie

Kuang Chao

Kaori Yoshida Coimbra

We would like to dedicate the collaborative effort of bringing a new edition of *Heat Transfer* to the memory of Prof. Donald K. Edwards, our teacher.

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<sup>1</sup> Books can be ordered directly at discounted prices at [www.temporalpublishing.com](http://www.temporalpublishing.com)

<sup>2</sup> [www.temporalpublishing.com/ht](http://www.temporalpublishing.com/ht)

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# PREFACE TO THE SECOND EDITION

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*Heat Transfer* has been written for students in mechanical, aerospace, nuclear, and chemical engineering programs. Apart from the usual lower-division mathematics and science courses, the preparation required of the student includes introductory courses in fluid mechanics and thermodynamics, and preferably the usual junior-level engineering mathematics course. The ordering of the material and the pace at which it is presented have been carefully chosen so that the beginning student can proceed from the most elementary concepts to those that are more difficult. As a result, the book should prove to be quite versatile. It can be used as the text for an introductory course during the junior or senior year, although the coverage is sufficiently comprehensive for use as a reference work in undergraduate laboratory and design courses, and by the practicing engineer.

Throughout the text, the emphasis is on engineering calculations, and each topic is developed to a point that will provide students with the tools needed to practice the art of design. The worked examples not only illustrate the use of relevant equations but also teach modeling as both an art and science. A supporting feature of *Heat Transfer* is the fully integrated software available from the author's website<sup>3</sup>. The software is intended to serve primarily as a tool for the student, both at college and after graduation in engineering practice. The programs are designed to reduce the effort required to obtain reliable numerical results and thereby increase the efficiency and effectiveness of the engineer. I have found the impact of the software on the educational process to be encouraging. It is now possible to assign more meaningful and interesting problems, because the students need not get bogged down in lengthy calculations. Parametric studies, which are the essence of engineering design, are relatively easily performed. Of course, computer programs are not a substitute for a proper understanding. The instructor is free to choose the extent to which the

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<sup>3</sup> <http://www.mae.ucla.edu/people/faculty/anthony-mills>

software is used by students because of the unique exact correspondence between the software and the text material. My practice has been to initially require students to perform various hand calculations, using the computer to give immediate feedback. For example, they do not have to wait a week or two until homework is returned to find that a calculated convective heat transfer coefficient was incorrect because a property table was misread.

The extent to which engineering design should be introduced in a heat transfer course is a controversial subject. It is my experience that students can be best introduced to design methodology through an increased focus on equipment such as heat exchangers: *Heat Transfer* presents more extensive coverage of exchanger design than do comparable texts. In the context of such equipment one can conveniently introduce topics such as synthesis, parametric studies, tradeoffs, optimization, economics, and material or health constraints. The computer program HEX2 assists the student to explore the consequences of changing the many parameters involved in the design process. If an appropriate selection of this material is taught, I am confident that Accreditation Board for Engineering and Technology guidelines for design content will be met. More important, I believe that engineering undergraduates are well served by being exposed to this material, even if it means studying somewhat less heat transfer science.

More than 300 new exercises have been added for this edition. They fall into two categories: (1) relatively straightforward exercises designed to help students understand fundamental concepts, and (2) exercises that introduce new technology and that have a practical flavor. The latter play a very important role in motivating students; considerable care has been taken to ensure that they are realistic in terms of parameter values and focus on significant aspects of real engineering problems. The practical exercises are first steps in the engineering design process and many have substantial design content. Since environmental considerations have required the phasing out of CFC refrigerants, R-12 and R-113 property data, worked examples and exercises, have been replaced with corresponding material for R-22 and R-134a.

*Heat Transfer* contains the following chapters and appendixes:

1. Introduction and Elementary Heat Transfer
2. Steady One-Dimensional Heat Conduction
3. Multidimensional and Unsteady Conduction
4. Convection Fundamentals and Correlations
5. Convection Analysis
6. Thermal Radiation
7. Condensation, Evaporation, and Boiling
8. Heat Exchangers
- A. Property Data
- B. Units, Conversion Factors, and Mathematics
- C. Charts

In a first course, the focus is always on the key topics of conduction, convection, radiation, and heat exchangers. Particular care has been taken to order the material on these topics from simpler to more difficult concepts. In Chapter 2 one-dimensional conduction and fins are treated before deriving the general partial differential heat conduction equation in Chapter 3. In Chapter 4 the student is taught how to use convection correlations before encountering the partial differential equations governing momentum and energy conservation in Chapter 5. In Chapter 6 radiation properties are introduced on a total energy basis and the shape factor is introduced as a geometrical concept to allow engineering problem solving before having to deal with the directional and spectral aspects of radiation. Also, wherever possible, advanced topics are located at the ends of chapters, and thus can be easily omitted in a first course.

Chapter 1 is a brief but self-contained introduction to heat transfer. Students are given an overview of the subject and some material needed in subsequent chapters. Interesting and relevant engineering problems can then be introduced at the earliest opportunity, thereby motivating student interest. All the exercises can be solved without accessing the property data in Appendix A.

Chapters 2 and 3 present a relatively conventional treatment of heat conduction, though the outdated and approximate Heissler and Gröber charts are replaced by exact charts and the computer program COND2. The treatment of finite-difference numerical methods for conduction has been kept concise and is based on finite-volume energy balances. Students are encouraged to solve the difference equations by writing their own computer programs, or by using standard mathematics software such as Mathcad or MATLAB.

In keeping with the overall philosophy of the book, the objective of Chapter 4 is to develop the students' ability to calculate convective heat transfer coefficients. The physics of convection is explained in a brief introduction, and the heat transfer coefficient is defined. Dimensional analysis using the Buckingham pi theorem is used to introduce the required dimensional groups and to allow a discussion of the importance of laboratory experiments. A large number of correlation formulas follow; instructors can discuss selected geometrical configurations as class time allows, and students can use the associated computer program CONV to reliably calculate heat transfer coefficients and skin friction coefficients or pressure drop for a wide range of configurations. Being able to do parametric studies with a wide variety of correlations enhances the students' understanding more than can be accomplished by hand calculations. Design alternatives can also be explored using CONV.

Analysis of convection is deferred to Chapter 5: simple laminar flows are considered, and high-speed flows are treated first in Section 5.2, since an understanding of the recovery temperature concept enhances the students' problem-solving capabilities. Each of the topics in Sections 5.3 through 5.8 are essentially self-contained, and the instructor can select as few or as many as required.

Chapter 6 focuses on thermal radiation. Radiation properties are initially defined on a total energy basis, and the shape factor is introduced as a simple geometrical concept. This approach allows students to immediately begin solving engineering radiation exchange problems. Only subsequently need they tackle the more difficult directional and spectral aspects of radiation. For gas radiation, the ubiquitous Hottel charts have been replaced by the more accurate methods developed by Edwards; the accompanying computer program RAD3 makes their use particularly simple.

The treatment of condensation and evaporation heat transfer in Chapter 7 has novel features, while the treatment of pool boiling is quite conventional. Forced convection boiling and condensation is taken far enough to facilitate calculation of both pressure drop and heat transfer. Heatpipes are dealt with in some detail, enabling students to calculate the wicking limit and to analyze the performance of simple gas-controlled heatpipes.

Chapter 8 expands the presentation of the thermal analysis of heat exchangers beyond the customary and includes regenerators and the effect of axial conduction on thermal performance. The treatment of heat exchanger design includes the calculation of exchanger pressure drop, thermal-hydraulic design, heat transfer surface selection for compact heat exchangers, and economic analysis leading to the calculation of the benefit-cost differential associated with heat recovery operations. The computer program HEX2 serves to introduce students to computer-aided design of heat exchangers.

The author and publisher appreciate the efforts of all those who provided input that helped develop and improve the text. We remain dedicated to further refining the text in future editions, and encourage you to contact us with any suggestions or comments you might have.

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Some of the material in *Heat Transfer*, in the form of examples and exercises, has been adapted from an earlier text by my former colleagues at UCLA, D. K. Edwards and V. E. Denny (*Transfer Processes* 1/e, Holt, Rinehart & Winston, 1973; 2/e Hemisphere-McGraw-Hill, 1979). I have also drawn on material in radiation heat transfer from a more recent text by D. K. Edwards (*Radiation Heat Transfer Notes*, Hemisphere, 1981). I gratefully acknowledge the contributions of these gentlemen, both to this book and to my professional career. The computer software was ably written by Baek Youn, Hae-Jin Choi, and Benjamin Tan. I would also like to thank former students S. W. Hiebert, R. Tsai, B. Cowan, E. Myhre, B. H. Chang, D. C. Weatherly, A. Gopinath, J. I. Rodriguez, B. P. Dooher, M. A. Friedman, and C. Yuen.

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# NOTES TO THE INSTRUCTOR AND STUDENT

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These notes have been prepared to assist the instructor and student and should be read before the text is used. Topics covered include conventions for artwork and mathematics, the format for example problems, organization of the exercises, comments on the thermophysical property data in Appendix A, and a guide for use of the accompanying computer software.

## ARTWORK

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Conventions used in the figures are as follows.

- Conduction or convection heat flow
- ↔ Radiation heat flow
- Fluid flow

## MATHEMATICAL SYMBOLS

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Symbols that may need clarification are as follows.

- $\simeq$  Nearly equal
- $\sim$  Of the same order of magnitude
- $|_x$  All quantities in the term to the left of the bar are evaluated at  $x$

## EXAMPLES

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Use of standard format for presenting the solutions of engineering problems is a good practice. The format used for the examples in *Heat Transfer*, which is but one possible approach, is as follows.

### **Problem statement**

### **Solution**

***Given:***

***Required:***

***Assumptions:*** 1.  
2. etc.

*Sketch* (when appropriate)

*Analysis* (diagrams when appropriate)

*Properties evaluation*

*Calculations*

*Results* (tables or graphs when appropriate)

### **Comments**

1.  
2. etc.

It is always assumed that the problem statement precedes the solution (as in the text) or that it is readily available (as in the *Solutions Manual*). Thus, the *Given* and *Required* statements are concise and focus on the essential features of the problem. Under *Assumptions*, the main assumptions required to solve the problem are listed; when appropriate, they are discussed further in the body of the solution. A sketch of the physical system is included when the geometry requires clarification; also, expected temperature and concentration profiles are given when appropriate. (Schematics that simply repeat the information in the problem statements are used sparingly. We know that many instructors always require a schematic. Our view is that students need to develop an appreciation of when a figure or graph is necessary, because artwork is usually an expensive component of engineering reports. For example, we see little use for a schematic that shows a 10 m length of straight 2 cm–O.D. tube.) The analysis may consist simply of listing some formulas from the text, or it may require setting up a differential equation and its solution. Strictly speaking, a property should not be evaluated until its need is identified by the analysis. However, in routine calculations, such as evaluation of convective heat transfer coefficients, it

is often convenient to list all the property values taken from an Appendix A table in one place. The calculations then follow with results listed, tabulated, or graphed as appropriate. Under *Comments*, the significance of the results can be discussed, the validity of assumptions further evaluated, or the broader implications of the problem noted.

In presenting calculations for the examples in *Heat Transfer*, we have rounded off results at each stage of the calculation. If additional figures are retained for the complete calculations, discrepancies in the last figure will be observed. Since many of the example calculations are quite lengthy, we believe our policy will facilitate checking a particular calculation step of concern. As is common practice, we have generally given results to more significant figures than is justified, so that these results can be conveniently used in further calculations. It is safe to say that no engineering heat transfer calculation will be accurate to within 1%, and that most experienced engineers will be pleased with results accurate to within 10% or 20%. Thus, preoccupation with a third or fourth significant figure is misplaced (unless required to prevent error magnification in operations such as subtraction). Fundamental constants are rounded off to no more than five significant figures.

## EXERCISES

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The diskette logo next to an exercise statement indicates that it can be solved using the *Heat Transfer* software, and that the sample solution provided to the instructor has been prepared accordingly. There are many additional exercises that can be solved using the software but that do not have the logo designation. These exercises are intended to give the student practice in hand calculations, and thus the sample solutions were also prepared manually.

The exercises have been ordered to correspond with the order in which the material is presented in the text, rather than in some increasing degree of difficulty. Since the range of difficulty of the exercises is considerable, the instructor is urged to give students guidance in selecting exercises for self-study. Answers to all exercises are listed in the *Solutions Manual* provided to instructors. Odd- and even-numbered exercises are listed separately; answers to odd-numbered exercises are available to students on the book website.

## PROPERTY DATA

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A considerable quantity of property data has been assembled in Appendix A. Key sources are given as references or are listed in the bibliography. Since *Heat Transfer* is a textbook, our primary objective in preparing Appendix A was to provide the student with a wide range of data in an easily used form. Whenever possible, we have used the most accurate data that we could obtain, but accuracy was not always the primary concern. For example, the need to have consistent data over a wide range of temperature often dictated the choice of source. All the tables are in SI units, with temperature in kelvins. The computer program UNITS can be used for

conversions to other systems of units. Appendix A should serve most needs of the student, as well as of the practicing engineer, for doing routine calculations. If a heat transfer research project requires accurate and reliable thermophysical property data, the prudent researcher should carefully check relevant primary data sources.

## SOFTWARE

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The HT software has a menu that describes the content of each program. The programs are also described at appropriate locations in the text. The input format and program use are demonstrated in example problems in the text. Use of the text index is recommended for locating the program descriptions and examples. There is a one-to-one correspondence between the text and the software. In principle, all numbers generated by the software can be calculated manually from formulas, graphs, and data given in the text. Small discrepancies may be seen when interpolation in graphs or property tables is required, since some of the data are stored in the software as polynomial curve fits.

The software facilitates self-study by the student. Practice hand calculations can be immediately checked using the software. When programs such as CONV, PHASE, and BOIL are used, properties evaluation and intermediate calculation steps can also be checked when the final results do not agree.

Since there is a large thermophysical property database stored in the software package, the programs can also be conveniently used to evaluate these properties for other purposes. For example, in CONV both the wall and fluid temperatures can be set equal to the desired temperature to obtain property values required for convection calculations. We can even go one step further when evaluating a convective heat transfer coefficient from a new correlation not contained in CONV: if a corresponding item is chosen, the values of relevant dimensionless groups can also be obtained from CONV, further simplifying the calculations.

Presently the HT software is available in both Windows and DOS versions. The latter can be used on both Mac OS X and Windows platforms with DOS emulators. Some examples in the text show sample inputs from the DOS version of the software. For the Windows version the inputs are essentially the same.

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# CONTENTS

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## CHAPTER

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<b>1</b>	<b>INTRODUCTION AND ELEMENTARY HEAT TRANSFER</b>	<b>1</b>
1.1	Introduction	2
1.2	Heat Transfer and Its Relation to Thermodynamics	3
1.3	Modes of Heat Transfer	7
1.3.1	Heat Conduction	8
1.3.2	Thermal Radiation	13
1.3.3	Heat Convection	17
1.4	Combined Modes of Heat Transfer	24
1.4.1	Thermal Circuits	24
1.4.2	Surface Energy Balances	27
1.5	Transient Thermal Response	29
1.5.1	The Lumped Thermal Capacity Model	29
1.5.2	Combined Convection and Radiation	34
1.6	Heat Exchangers	37
1.6.1	Single- and Two-Stream Exchangers	38
1.6.2	Analysis of a Condenser	40
1.6.3	Other Single-Stream Exchangers	45
1.7	Dimensions and Units	45
1.8	Closure	47
	Exercises	48
<b>2</b>	<b>STEADY ONE-DIMENSIONAL HEAT CONDUCTION</b>	<b>67</b>
2.1	Introduction	68
2.2	Fourier's Law of Heat Conduction	68
2.2.1	Thermal Conductivity	69
2.2.2	Contact Resistance	71
2.3	Conduction Across Cylindrical and Spherical Shells	73
2.3.1	Conduction across a Cylindrical Shell	73
2.3.2	Critical Thickness of Insulation on a Cylinder	77
2.3.3	Conduction across a Spherical Shell	80

2.3.4	Conduction with Internal Heat Generation	82
<b>2.4</b>	<b>Fins</b>	<b>86</b>
2.4.1	The Pin Fin	86
2.4.2	Fin Resistance and Surface Efficiency	94
2.4.3	Other Fin Type Analyses	95
2.4.4	Fins of Varying Cross-Sectional Area	100
2.4.5	The Similarity Principle and Dimensional Analysis	108
<b>2.5</b>	<b>Closure</b>	<b>111</b>
	<b>References</b>	<b>112</b>
	<b>Exercises</b>	<b>112</b>
 <b>3</b>	 <b>MULTIDIMENSIONAL AND UNSTEADY CONDUCTION</b>	 <b>143</b>
<b>3.1</b>	<b>Introduction</b>	<b>144</b>
<b>3.2</b>	<b>The Heat Conduction Equation</b>	<b>144</b>
3.2.1	Fourier's Law as a Vector Equation	145
3.2.2	Derivation of the Heat Conduction Equation	145
3.2.3	Boundary and Initial Conditions	150
3.2.4	Solution Methods	153
<b>3.3</b>	<b>Multidimensional Steady Conduction</b>	<b>154</b>
3.3.1	Steady Conduction in a Rectangular Plate	154
3.3.2	Steady Conduction in a Rectangular Block	161
3.3.3	Conduction Shape Factors	164
<b>3.4</b>	<b>Unsteady Conduction</b>	<b>167</b>
3.4.1	The Slab with Negligible Surface Resistance	168
3.4.2	The Semi-Infinite Solid	175
3.4.3	Convective Cooling of Slabs, Cylinders, and Spheres	187
3.4.4	Product Solutions for Multidimensional Unsteady Conduction	198
<b>3.5</b>	<b>Moving-Boundary Problems</b>	<b>203</b>
3.5.1	Solidification from a Melt	203
3.5.2	Steady-State Melting Ablation	207
<b>3.6</b>	<b>Numerical Solution Methods</b>	<b>212</b>
3.6.1	A Finite-Difference Method for Two-Dimensional Steady Conduction	213
3.6.2	Finite-Difference Methods for One-Dimensional Unsteady Conduction	221
3.6.3	Resistance-Capacitance ( $RC$ ) Formulation	230
3.6.4	A Finite-Difference Method for Moving-Boundary Problems	237
<b>3.7</b>	<b>Closure</b>	<b>242</b>
	<b>References</b>	<b>243</b>
	<b>Exercises</b>	<b>244</b>
 <b>4</b>	 <b>CONVECTION FUNDAMENTALS AND CORRELATIONS</b>	 <b>275</b>
<b>4.1</b>	<b>Introduction</b>	<b>276</b>
<b>4.2</b>	<b>Fundamentals</b>	<b>276</b>
4.2.1	The Convective Heat Transfer Coefficient	277
4.2.2	Dimensional Analysis	283

4.2.3	Correlation of Experimental Data	295
4.2.4	Evaluation of Fluid Properties	299
<b>4.3</b>	<b>Forced Convection</b>	<b>301</b>
4.3.1	Forced Flow in Tubes and Ducts	301
4.3.2	External Forced Flows	312
<b>4.4</b>	<b>Natural Convection</b>	<b>325</b>
4.4.1	External Natural Flows	325
4.4.2	Internal Natural Flows	333
4.4.3	Mixed Forced and Natural Flows	340
<b>4.5</b>	<b>Tube Banks and Packed Beds</b>	<b>347</b>
4.5.1	Flow through Tube Banks	348
4.5.2	Flow through Packed Beds	355
<b>4.6</b>	<b>Rotating Surfaces</b>	<b>362</b>
4.6.1	Rotating Disks, Spheres, and Cylinders	362
<b>4.7</b>	<b>Rough Surfaces</b>	<b>365</b>
4.7.1	Effect of Surface Roughness	366
<b>4.8</b>	<b>The Computer Program CONV</b>	<b>375</b>
<b>4.9</b>	<b>Closure</b>	<b>375</b>
	References	384
	Exercises	387
<b>5</b>	<b>CONVECTION ANALYSIS</b>	<b>413</b>
<b>5.1</b>	<b>Introduction</b>	<b>414</b>
<b>5.2</b>	<b>High-Speed Flows</b>	<b>415</b>
5.2.1	A Couette Flow Model	415
5.2.2	The Recovery Factor Concept	420
<b>5.3</b>	<b>Laminar Flow in a Tube</b>	<b>422</b>
5.3.1	Momentum Transfer in Hydrodynamically Fully Developed Flow	423
5.3.2	Fully Developed Heat Transfer for a Uniform Wall Heat Flux	426
<b>5.4</b>	<b>Laminar Boundary Layers</b>	<b>432</b>
5.4.1	The Governing Equations for Forced Flow along a Flat Plate	433
5.4.2	The Plug Flow Model	435
5.4.3	Integral Solution Method	437
5.4.4	Self-Similar Solutions	446
5.4.5	Natural Convection on an Isothermal Vertical Wall	455
<b>5.5</b>	<b>Turbulent Flows</b>	<b>461</b>
5.5.1	The Prandtl Mixing Length and the Eddy Diffusivity Model	462
5.5.2	Forced Flow along a Flat Plate	465
5.5.3	Flow in a Tube	478
5.5.4	More Advanced Turbulence Models	486
<b>5.6</b>	<b>Similarity and Modeling</b>	<b>486</b>
5.6.1	Dimensionless Equations and Boundary Conditions	486
5.6.2	Modeling	492
<b>5.7</b>	<b>The General Conservation Equations</b>	<b>493</b>
5.7.1	Conservation of Mass	493
5.7.2	Conservation of Momentum	495

- 5.7.3 Conservation of Energy 499
- 5.7.4 Use of the Conservation Equations 503
- 5.8 Scale Analysis 504**
  - 5.8.1 Forced-Convection Laminar Boundary Layers 504
  - 5.8.2 Natural-Convection Laminar Boundary Layer on a Vertical Wall 510
- 5.9 Closure 515**
  - References 516**
  - Exercises 517**

## **6 THERMAL RADIATION 531**

- 6.1 Introduction 532**
- 6.2 The Physics of Radiation 532**
  - 6.2.1 The Electromagnetic Spectrum 533
  - 6.2.2 The Black Surface 534
  - 6.2.3 Real Surfaces 536
- 6.3 Radiation Exchange Between Surfaces 538**
  - 6.3.1 Radiation Exchange between Black Surfaces 538
  - 6.3.2 Shape Factors and Shape Factor Algebra 540
  - 6.3.3 Electrical Network Analogy for Black Surfaces 547
  - 6.3.4 Radiation Exchange between Two Diffuse Gray Surfaces 550
  - 6.3.5 Radiation Exchange between Many Diffuse Gray Surfaces 557
  - 6.3.6 Radiation Transfer through Passages 565
- 6.4 Solar Radiation 568**
  - 6.4.1 Solar Irradiation 568
  - 6.4.2 Atmospheric Radiation 570
  - 6.4.3 Solar Absorptance and Transmittance 572
- 6.5 Directional Characteristics of Surface Radiation 577**
  - 6.5.1 Radiation Intensity and Lambert's Law 578
  - 6.5.2 Shape Factor Determination 581
  - 6.5.3 Directional Properties of Real Surfaces 584
- 6.6 Spectral Characteristics of Surface Radiation 590**
  - 6.6.1 Planck's Law and Fractional Functions 590
  - 6.6.2 Spectral Properties 593
- 6.7 Radiation Transfer Through Gases 599**
  - 6.7.1 The Equation of Transfer 600
  - 6.7.2 Gas Radiation Properties 601
  - 6.7.3 Effective Beam Lengths for an Isothermal Gas 609
  - 6.7.4 Radiation Exchange between an Isothermal Gas and a Black Enclosure 614
  - 6.7.5 Radiation Exchange between an Isothermal Gray Gas and a Gray Enclosure 615
  - 6.7.6 Radiation Exchange between an Isothermal Nongray Gas and a Single-Gray-Surface Enclosure 619
- 6.8 Closure 621**
  - References 622**
  - Exercises 623**

<b>7</b>	<b>CONDENSATION, EVAPORATION, AND BOILING</b>	<b>651</b>
7.1	<b>Introduction</b>	<b>652</b>
7.2	<b>Film Condensation</b>	<b>652</b>
7.2.1	Laminar Film Condensation on a Vertical Wall	654
7.2.2	Wavy Laminar and Turbulent Film Condensation on a Vertical Wall	662
7.2.3	Laminar Film Condensation on Horizontal Tubes	668
7.2.4	Effects of Vapor Velocity and Vapor Superheat	674
7.3	<b>Film Evaporation</b>	<b>681</b>
7.3.1	Falling Film Evaporation on a Vertical Wall	681
7.4	<b>Pool Boiling</b>	<b>685</b>
7.4.1	Regimes of Pool Boiling	685
7.4.2	Boiling Inception	688
7.4.3	Nucleate Boiling	691
7.4.4	The Peak Heat Flux	693
7.4.5	Film Boiling	696
7.5	<b>Forced-Convection Boiling and Condensation</b>	<b>702</b>
7.5.1	Two-phase Flow Patterns	702
7.5.2	Pressure Drop	708
7.5.3	Internal Forced-Convection Boiling	712
7.5.4	Internal Forced-Convection Condensation	716
7.6	<b>Phase Change At Low Pressures</b>	<b>719</b>
7.6.1	Kinetic Theory of Phase Change	719
7.6.2	Interfacial Heat Transfer Resistance	723
7.6.3	Nusselt Analysis Including Interfacial Resistance	725
7.6.4	Engineering Significance of the Interfacial Resistance	728
7.7	<b>Heatpipes</b>	<b>730</b>
7.7.1	Capillary Pumping	733
7.7.2	Sonic, Entrainment, and Boiling Limitations	738
7.7.3	Gas-Loaded Heatpipes	740
7.8	<b>Closure</b>	<b>744</b>
	<b>References</b>	<b>745</b>
	<b>Exercises</b>	<b>748</b>
<b>8</b>	<b>HEAT EXCHANGERS</b>	<b>761</b>
8.1	<b>Introduction</b>	<b>762</b>
8.2	<b>Types of Heat Exchangers</b>	<b>762</b>
8.2.1	Geometric Flow Configurations	764
8.2.2	Fluid Temperature Behavior	767
8.2.3	Heat Transfer Surfaces	769
8.2.4	Direct-Contact Exchangers	769
8.3	<b>Energy Balances and Overall Heat Transfer Coefficients</b>	<b>770</b>
8.3.1	Exchanger Energy Balances	770
8.3.2	Overall Heat Transfer Coefficients	772
8.4	<b>Single-stream Steady-flow Heat Exchangers</b>	<b>777</b>
8.4.1	Analysis of an Evaporator	778

<b>8.5</b>	<b>Two-stream Steady-flow Heat Exchangers</b>	<b>781</b>
8.5.1	The Logarithmic Mean Temperature Difference	781
8.5.2	Effectiveness and Number of Transfer Units	786
8.5.3	Balanced-Flow Exchangers	794
8.5.4	Effect of Axial Conduction	797
<b>8.6</b>	<b>Regenerators</b>	<b>805</b>
8.6.1	Balanced Counterflow Regenerators	805
<b>8.7</b>	<b>Elements of Heat Exchanger Design</b>	<b>811</b>
8.7.1	Exchanger Pressure Drop	813
8.7.2	Thermal-Hydraulic Exchanger Design	820
8.7.3	Surface Selection for Compact Heat Exchangers	827
8.7.4	Economic Analysis	830
8.7.5	Computer-Aided Heat Exchanger Design: HEX2	835
<b>8.8</b>	<b>Closure</b>	<b>846</b>
	<b>References</b>	<b>847</b>
	<b>Exercises</b>	<b>848</b>

## APPENDIX

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### A PROPERTY DATA 871

<b>Table A.1a</b>	Solid metals: Melting point and thermal properties at 300 K	873
<b>Table A.1b</b>	Solid metals: Temperature dependence of thermal conductivity	875
<b>Table A.1c</b>	Solid metals: Temperature dependence of specific heat capacity	876
<b>Table A.2</b>	Solid dielectrics: Thermal properties	877
<b>Table A.3</b>	Insulators and building materials: Thermal properties	879
<b>Table A.4</b>	Thermal conductivity of selected materials at cryogenic temperatures	881
<b>Table A.5a</b>	Total hemispherical emittance at $T_s \simeq 300$ K, and solar absorptance	882
<b>Table A.5b</b>	Temperature variation of total hemispherical emittance for selected surfaces	885
<b>Table A.6a</b>	Spectral and total absorptances of metals for normal incidence	886
<b>Table A.6b</b>	Spectral absorptances at room temperature and an angle of incidence of $25^\circ$ from the normal	887
<b>Table A.7</b>	Gases: Thermal properties	888
<b>Table A.8</b>	Dielectric liquids: Thermal properties	892
<b>Table A.9</b>	Liquid metals: Thermal properties	895
<b>Table A.10a</b>	Volume expansion coefficients for liquids	896
<b>Table A.10b</b>	Density and volume expansion coefficient of water	897
<b>Table A.11</b>	Surface tensions in contact with air	898
<b>Table A.12a</b>	Thermodynamic properties of saturated steam	899
<b>Table A.12b</b>	Thermodynamic properties of saturated ammonia	902
<b>Table A.12c</b>	Thermodynamic properties of saturated nitrogen	903

<b>Table A.12d</b>	Thermodynamic properties of saturated mercury	904
<b>Table A.12e</b>	Thermodynamic properties of saturated refrigerant-22	905
<b>Table A.12f</b>	Thermodynamic properties of saturated refrigerant-134a	906
<b>Table A.13a</b>	Aqueous ethylene glycol solutions: Thermal properties	907
<b>Table A.13b</b>	Aqueous sodium chloride solutions: Thermal properties	908
<b>Table A.14a</b>	Dimensions of commercial pipes [mm] (ASA standard)	909
<b>Table A.14b</b>	Dimensions of commercial tubes [mm] (ASTM standard)	910
<b>Table A.14c</b>	Dimensions of seamless steel tubes for tubular heat exchangers [mm] (DIN 28 180)	911
<b>Table A.14d</b>	Dimensions of wrought copper and copper alloy tubes for condensers and heat exchangers [mm] (DIN 1785-83)	911
<b>Table A.14e</b>	Dimensions of seamless cold drawn stainless steel tubes [mm] (LN 9398)	912
<b>Table A.14f</b>	Dimensions of seamless drawn wrought aluminum alloy tubes [mm] (LN 9223)	912
<b>Table A.15</b>	U.S. standard atmosphere	913
<b>Table A.16</b>	Selected physical constants	914

## **B UNITS, CONVERSION FACTORS, AND MATHEMATICS 915**

<b>Table B.1a</b>	Base and supplementary SI units	916
<b>Table B.1b</b>	Derived SI units	916
<b>Table B.1c</b>	Recognized non-SI units	917
<b>Table B.1d</b>	Multiples of SI units	917
<b>Table B.2</b>	Conversion factors	918
<b>Table B.3</b>	Bessel functions	919
<b>Table B.3a</b>	Bessel functions of the first and second kinds	920
<b>Table B.3b</b>	Modified Bessel functions of the first and second kinds	922
<b>Table B.4</b>	The complementary error function	924

## **C CHARTS 925**

<b>Figure C.1a</b>	Centerplane temperature response for a convectively cooled slab; $Bi = h_c L/k$ , where $L$ is the slab half-width	926
<b>Figure C.1b</b>	Centerline temperature response for a convectively cooled cylinder; $Bi = h_c R/k$	927
<b>Figure C.1c</b>	Center temperature response for a convectively cooled sphere; $Bi = h_c R/k$	927
<b>Figure C.2a</b>	Fractional energy loss for a convectively cooled slab; $Bi = h_c L/k$ , where $L$ is the slab half-width	928
<b>Figure C.2b</b>	Fractional energy loss for a convectively cooled cylinder; $Bi = h_c R/k$	928
<b>Figure C.2c</b>	Fractional energy loss for a convectively cooled sphere; $Bi = h_c R/k$	929
<b>Figure C.3a</b>	Shape (view) factor for coaxial parallel disks	929
<b>Figure C.3b</b>	Shape (view) factor for opposite rectangles	930

<b>Figure C.3c</b>	Shape (view) factor for adjacent rectangles	930
<b>Figure C.4a</b>	LMTD correction factor for a heat exchanger with one shell pass and 2, 4, 6, ... tube passes	931
<b>Figure C.4b</b>	LMTD correction factor for a cross-flow heat exchanger with both fluids unmixed	931
<b>Figure C.4c</b>	LMTD correction factor for a cross-flow heat exchanger with both fluids mixed	932
<b>Figure C.4d</b>	LMTD correction factor for a cross-flow heat exchanger with two tube passes (unmixed) and one shell pass (mixed)	932

<b>Bibliography</b>	<b>933</b>
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<b>Nomenclature</b>	<b>941</b>
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<b>Index</b>	<b>947</b>
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# INTRODUCTION AND ELEMENTARY HEAT TRANSFER

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CONTENTS

- 1.1 INTRODUCTION
- 1.2 HEAT TRANSFER AND ITS RELATION TO THERMODYNAMICS
- 1.3 MODES OF HEAT TRANSFER
- 1.4 COMBINED MODES OF HEAT TRANSFER
- 1.5 TRANSIENT THERMAL RESPONSE
- 1.6 HEAT EXCHANGERS
- 1.7 DIMENSIONS AND UNITS
- 1.8 CLOSURE

## 1.1 INTRODUCTION

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The process of heat transfer is familiar to us all. On a cold day we put on more clothing to reduce heat transfer from our warm body to cold surroundings. To make a cup of coffee we may plug in a kettle, inside which heat is transferred from an electrical resistance element to the water, heating the water until it boils. The engineering discipline of **heat transfer** is concerned with methods of calculating **rates** of heat transfer. These methods are used by engineers to design components and systems in which heat transfer occurs. Heat transfer considerations are important in almost all areas of technology. Traditionally, however, the discipline that has been most concerned with heat transfer is mechanical engineering because of the importance of heat transfer in energy conversion systems, from coal-fired power plants to solar water heaters.

Many *thermal design* problems require reducing heat transfer rates by providing suitable *insulation*. The insulation of buildings in extreme climates is a familiar example, but there are many others. The space shuttle has thermal tiles to insulate the vehicle from high-temperature air behind the bow shock wave during reentry into the atmosphere. Cryostats, which maintain the cryogenic temperatures required for the use of superconductors, must be effectively insulated to reduce the cooling load on the refrigeration system. Often, the only way to ensure protection from severe heating is to provide a fluid flow as a heat “sink”. Nozzles of liquid-fueled rocket motors are cooled by pumping the cold fuel through passages in the nozzle wall before injection into the combustion chamber. A critical component in a fusion reactor is the “first wall” of the containment vessel, which must withstand intense heating from the hot plasma. Such walls may be cooled by a flow of helium gas or liquid lithium.

A common thermal design problem is the transfer of heat from one fluid to another. Devices for this purpose are called *heat exchangers*. A familiar example is the automobile radiator, in which heat is transferred from the hot engine coolant to cold air blowing through the radiator core. Heat exchangers of many different types are required for power production and by the process industries. A power plant, whether the fuel be fossil or nuclear, has a *boiler* in which water is evaporated to produce steam to drive the turbines, and a *condenser* in which the steam is condensed to provide a low back pressure on the turbines and for water recovery. The condenser patented by James Watt in 1769 more than doubled the efficiency of steam engines then being used and set the Industrial Revolution in motion. The common vapor cycle refrigeration or air-conditioning system has an *evaporator* where heat is absorbed at low temperature and a *condenser* where heat is rejected at a higher temperature. On a domestic refrigerator, the condenser is usually in the form of a tube coil with cooling *fins* to assist transfer of heat to the surroundings. An oil refinery has a great variety of heat transfer equipment, including rectification columns and thermal crackers. Many heat exchangers are used to transfer heat from one process stream to another, to reduce the total energy consumption by the refinery.

Often the design problem is one of *thermal control*, that is, maintaining the operating temperature of temperature-sensitive components within a specified range.

Cooling of all kinds of electronic gear is an example of thermal control. The development of faster computers is now severely constrained by the difficulty of controlling the temperature of very small components, which dissipate large amounts of heat. Thermal control of temperature-sensitive components in a communications satellite orbiting the Earth is a particularly difficult problem. Transistors and diodes must not overheat, batteries must not freeze, telescope optics must not lose alignment due to thermal expansion, and photographs must be processed at the proper temperature to ensure high resolution. Thermal control of space stations present even greater problems, since reliable life-support systems are also necessary.

From the foregoing examples, it is clear that heat transfer involves a great variety of physical phenomena and engineering systems. The phenomena must first be understood and quantified before a methodology for the thermal design of an engineering system can be developed. Chapter 1 is an overview of the subject and introduces key topics at an elementary level. In Section 1.2, the distinction between the subjects of heat transfer and thermodynamics is explained. The first law of thermodynamics is reviewed, and closed- and open-system forms required for heat transfer analysis are developed. Section 1.3 introduces the three important modes of heat transfer: **heat conduction**, **thermal radiation**, and **heat convection**. Some formulas are developed that allow elementary heat transfer calculations to be made. In practical engineering problems, these modes of heat transfer usually occur simultaneously. Thus, in Section 1.4, the analysis of heat transfer by combined modes is introduced. Engineers are concerned with the changes heat transfer processes effect in engineering systems and, in Section 1.5, an example is given in which the first law is applied to a simple model closed system to determine the temperature response of the system with time. Finally, in Section 1.6, the International System of units (SI) is reviewed, and the units policy that is followed in the text is discussed.

## 1.2 HEAT TRANSFER AND ITS RELATION TO THERMODYNAMICS

---

When a hot object is placed in cold surroundings, it cools: the object loses internal energy, while the surroundings gain internal energy. We commonly describe this interaction as a *transfer of heat* from the object to the surrounding region. Since the caloric theory of heat has been long discredited, we do not imagine a “heat substance” flowing from the object to the surroundings. Rather, we understand that internal energy has been transferred by complex interactions on an atomic or subatomic scale. Nevertheless, it remains common practice to describe these interactions as transfer, transport, or flow, of heat. The engineering discipline of heat transfer is concerned with calculation of the rate at which heat flows within a medium, across an interface, or from one surface to another, as well as with the calculation of associated temperatures.

It is important to understand the essential difference between the engineering discipline of heat transfer and what is commonly called thermodynamics. Classical thermodynamics deals with systems in equilibrium. Its methodology may be used

to calculate the energy required to change a system from one equilibrium state to another, but it cannot be used to calculate the rate at which the change may occur. For example, if a 1 kg ingot of iron is quenched from 1000°C to 100°C in an oil bath, thermodynamics tells us that the loss in internal energy of the ingot is mass (1 kg)  $\times$  specific heat capacity ( $\sim 450$  J/kg K)  $\times$  temperature change (900 K), or approximately 405 kJ. But thermodynamics cannot tell us how long we will have to wait for the temperature to drop to 100°C. The time depends on the temperature of the oil bath, physical properties of the oil, motion of the oil, and other factors. An appropriate heat transfer analysis will consider all of these.

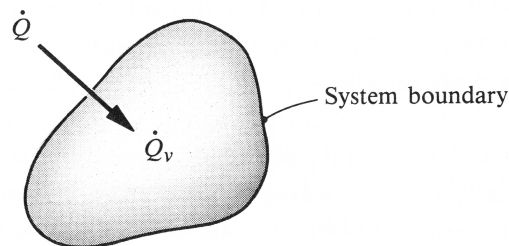
Analysis of heat transfer processes does require using some thermodynamics concepts. In particular, the **first law of thermodynamics** is used, generally in particularly simple forms since work effects can often be ignored. The first law is a statement of the *principle of conservation of energy*, which is a basic law of physics. This principle can be formulated in many ways by excluding forms of energy that are irrelevant to the problem under consideration, or by simply redefining what is meant by energy. In heat transfer, it is common practice to refer to the first law as the *energy conservation principle* or simply as an *energy* or *heat balance* when no work is done. However, as in thermodynamics, it is essential that the correct form of the first law be used. The student must be able to define an appropriate system, recognize whether the system is *open* or *closed*, and decide whether a steady state can be assumed. Some simple forms of the energy conservation principle, which find frequent use in this text, follow.

A closed system containing a fixed mass of a solid is shown in Fig. 1.1. The system has a volume  $V[\text{m}^3]$ , and the solid has a density  $\rho [\text{kg}/\text{m}^3]$ . There is *net* heat transfer into the system at a rate of  $\dot{Q} [\text{J}/\text{s}$  or  $\text{W}]$ , and heat may be generated within the solid, for example, by nuclear fission or by an electrical current, at a rate  $\dot{Q}_v [\text{W}]$ . Solids may be taken to be incompressible, so no work is done by or on the system. The principle of conservation of energy requires that over a time interval  $\Delta t$  [s],

$$\begin{aligned} \text{Change in internal energy} &= \text{Net heat transferred} + \text{Heat generated} \\ \text{within the system} &\quad \text{into the system} \quad \text{within the system} \\ \Delta U &= \dot{Q}\Delta t + \dot{Q}_v\Delta t \end{aligned} \quad (1.1)$$

Dividing by  $\Delta t$  and letting  $\Delta t$  go to zero gives

$$\frac{dU}{dt} = \dot{Q} + \dot{Q}_v$$

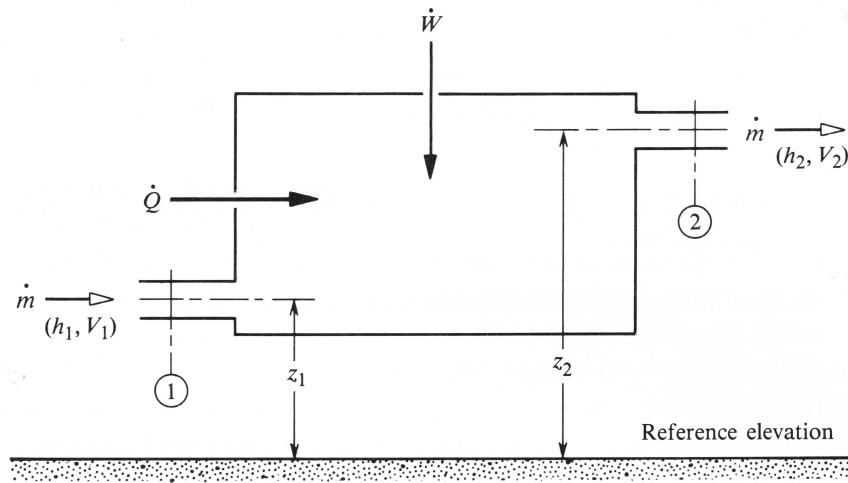


**Figure 1.1** Application of the energy conservation principle to a closed system.

The system contains a fixed mass ( $\rho V$ ); thus, we can write  $dU = \rho V du$ , where  $u$  is the specific internal energy [J/kg]. Also, for an incompressible solid,  $du = c_v dT$ , where  $c_v$  is the constant-volume specific heat<sup>1</sup> [J/kg K], and  $T$  [K] is temperature. Since the solid has been taken to be incompressible, the constant-volume and constant-pressure specific heats are equal, so we simply write  $du = cdT$  to obtain

$$\rho V c \frac{dT}{dt} = \dot{Q} + \dot{Q}_v \quad (1.2)$$

Equation (1.2) is a special form of the first law of thermodynamics that will be used often in this text. It is written on a *rate* basis; that is, it gives the rate of change of temperature with time. For some purposes, however, it will prove convenient to return to Eq. (1.1) as a statement of the first law.



**Figure 1.2** Application of the energy conservation principle to a steady-flow open system.

Figure 1.2 shows an *open* system (or *control volume*), for which a useful form of the first law is the **steady-flow energy equation**. It is used widely in the thermodynamic analysis of equipment such as turbines and compressors. Then

$$\dot{m} \Delta \left( h + \frac{V^2}{2} + gz \right) = \dot{Q} + \dot{W} \quad (1.3)$$

where  $\dot{m}$  [kg/s] is the mass flow rate,  $h$  [J/kg] is the specific enthalpy,  $V$  [m/s] is velocity,  $g$  [m/s<sup>2</sup>] is the gravitational acceleration,  $z$  is elevation [m],  $\dot{Q}$  [W] is the net rate of heat transfer, as before, and  $\dot{W}$  [W] is the rate at which external (shaft) work is done on the system.<sup>2</sup> Notice that the sign convention here is that external work done *on* the system is positive; the opposite sign convention is also widely used. The symbol  $\Delta X$  means  $X_{\text{out}} - X_{\text{in}}$ , or the change in  $X$ . Equation (1.3) applies to a pure

<sup>1</sup> The terms *specific heat capacity* and *specific heat* are equivalent and interchangeable in the heat transfer literature.

<sup>2</sup> Equation (1.3) has been written as if  $h$ ,  $V$ , and  $z$  are uniform in the streams crossing the control volume boundary. Often such an assumption can be made; if not, an integration across each stream is required to give appropriate average values.

substance when conditions within the system, such as temperature and velocity, are unchanging over some appropriate time interval. Heat generation within the system has not been included. In many types of heat transfer equipment, no external work is done, and changes in kinetic and potential energy are negligible; Eq. (1.3) then reduces to

$$\dot{m}\Delta h = \dot{Q} \quad (1.4)$$

The specific enthalpy  $h$  is related to the specific internal energy  $u$  as

$$h = u + Pv \quad (1.5)$$

where  $P$  [N/m<sup>2</sup> or Pa] is pressure, and  $v$  is specific volume [m<sup>3</sup>/kg]. Two limit forms of  $\Delta h$  are useful. If the fluid enters the system at state 1 and leaves at state 2:

1. For ideal gases with  $Pv = RT$ ,

$$\Delta h = \int_{T_1}^{T_2} c_p dT \quad (1.6a)$$

where  $R$  [J/kg K] is the gas constant and  $c_p$  [J/kg K] is the constant-pressure specific heat.

2. For incompressible liquids with  $\rho = 1/v = \text{constant}$

$$\Delta h = \int_{T_1}^{T_2} c dT + \frac{P_2 - P_1}{\rho} \quad (1.6b)$$

where  $c = c_v = c_p$ . The second term in Eq. (1.6b) is often negligible as will be assumed throughout this text.

Equation (1.4) is the usual starting point for the heat transfer analysis of steady-state open systems.

The *second law of thermodynamics* tells us that if two objects at temperatures  $T_1$  and  $T_2$  are connected, and if  $T_1 > T_2$ , then heat will flow spontaneously and irreversibly from object 1 to object 2. Also, there is an entropy increase associated with this heat flow. As  $T_2$  approaches  $T_1$ , the process approaches a reversible process, but simultaneously the rate of heat transfer approaches zero, so the process is of little practical interest. All heat transfer processes encountered in engineering are irreversible and generate entropy. With the increasing realization that energy supplies should be conserved, efficient use of available energy is becoming an important consideration in thermal design. Thus, the engineer should be aware of the irreversible processes occurring in the system under development and understand that the optimal design may be one that minimizes entropy generation due to heat transfer and fluid flow. Most often, however, energy conservation is simply a consideration in the overall economic evaluation of the design. Usually there is an important trade-off between energy costs associated with the operation of the system and the capital costs required to construct the equipment.

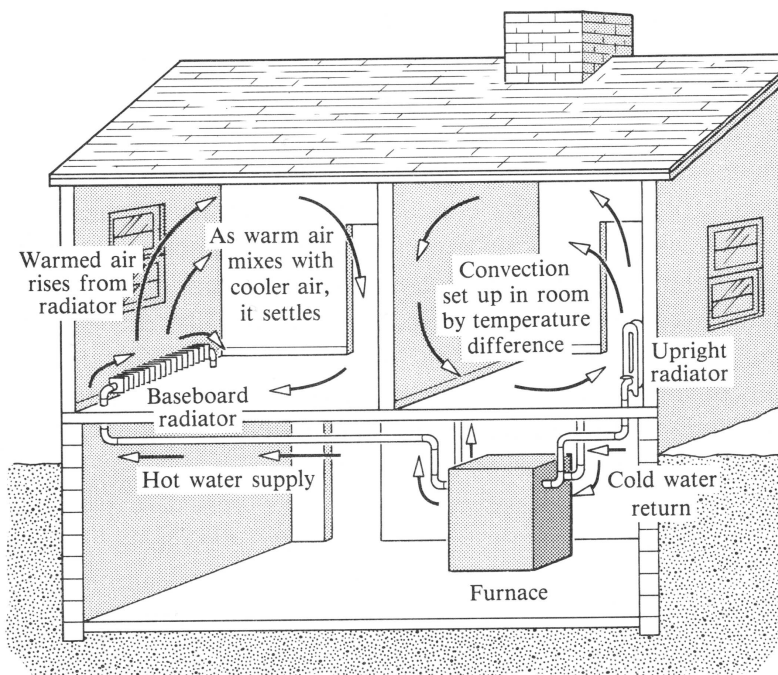
### 1.3 MODES OF HEAT TRANSFER

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In thermodynamics, *heat* is defined as energy transfer due to temperature gradients or differences. Consistent with this viewpoint, thermodynamics recognizes only two modes of heat transfer: *conduction* and *radiation*. For example, heat transfer across a steel pipe wall is by conduction, whereas heat transfer from the Sun to the Earth or to a spacecraft is by thermal radiation. These modes of heat transfer occur on a molecular or subatomic scale. In air at normal pressure, conduction is by molecules that travel a very short distance ( $\sim 0.065\mu\text{m}$ ) before colliding with another molecule and exchanging energy. On the other hand, radiation is by photons, which travel almost unimpeded through the air from one surface to another. Thus, an important distinction between conduction and radiation is that the energy carriers for conduction have a short *mean free path*, whereas for radiation the carriers have a long mean free path. However, in air at the very low pressures characteristic of high-vacuum equipment, the mean free path of molecules can be much longer than the equipment dimensions, so the molecules travel unimpeded from one surface to another. Then heat transfer by molecules is governed by laws analogous to those for radiation.

A fluid, by virtue of its mass and velocity, can transport momentum. In addition, by virtue of its temperature, it can transport energy. Strictly speaking, *convection* is the transport of energy by bulk motion of a medium (a moving solid can also convect energy in this sense). In the steady-flow energy equation, Eq. (1.3), convection of internal energy is contained in the term  $\dot{m}\Delta h$ , which is on the left-hand side of the equation, and heat transfer by conduction and radiation is on the right-hand side, as  $\dot{Q}$ . However, it is common engineering practice to use the term *convection* more broadly and describe heat transfer from a surface to a moving fluid also as convection, or *convective heat transfer*, even though conduction and radiation play a dominant role close to the surface, where the fluid is stationary. In this sense, convection is usually regarded as a distinct mode of heat transfer. Examples of convective heat transfer include heat transfer from the radiator of an automobile or to the skin of a hypersonic vehicle. Convection is often associated with a change of phase, for example, when water boils in a kettle or when steam condenses in a power plant condenser. Owing to the complexity of such processes, boiling and condensation are often regarded as distinct heat transfer processes.

The hot water home heating system shown in Fig. 1.3 illustrates the modes of heat transfer. Hot water from the furnace in the basement flows along pipes to radiators located in individual rooms. Transport of energy by the hot water from the basement is true convection as defined above; we do not call this a heat transfer process. Inside the radiators, there is convective heat transfer from the hot water to the radiator shell, conduction across the radiator shell, and both convective and radiative heat transfer from the hot outer surface of the radiator shell into the room. The convection is *natural* convection: the heated air adjacent to the radiator surface rises due to its buoyancy, and cooler air flows in to take its place. The radiators are heat exchangers. Although commonly used, the term *radiator* is misleading since heat transfer



**Figure 1.3** A hot-water home heating system illustrating the modes of heat transfer.

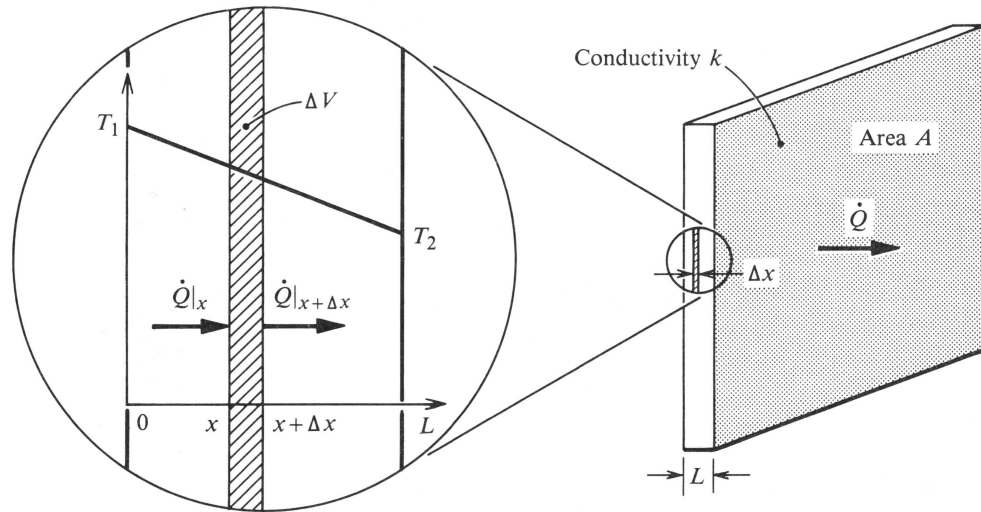
from the shell surface can be predominantly by convection rather than by radiation (see Exercise 1–20). Heaters that transfer heat predominantly by radiation are, for example, electrical resistance wire units.

Each of the three important subject areas of heat transfer will now be introduced: conduction, in Section 1.3.1; radiation, in Section 1.3.2; and convection, in Section 1.3.3.

### 1.3.1 Heat Conduction

On a microscopic level, the physical mechanisms of conduction are complex, encompassing such varied phenomena as molecular collisions in gases, lattice vibrations in crystals, and flow of free electrons in metals. However, if at all possible, the engineer avoids considering processes at the microscopic level, preferring to use *phenomenological laws*, at a macroscopic level. The phenomenological law governing heat conduction was proposed by the French mathematical physicist J. B. Fourier in 1822. This law will be introduced here by considering the simple problem of one-dimensional heat flow across a plane wall—for example, a layer of insulation.<sup>3</sup> Figure 1.4 shows a plane wall of surface area  $A$  and thickness  $L$ , with its face at  $x = 0$  maintained at temperature  $T_1$  and the face at  $x = L$  maintained at  $T_2$ . The heat flow  $\dot{Q}$  through the wall is in the direction of decreasing temperature: if

<sup>3</sup> In thermodynamics, the term *insulated* is often used to refer to a *perfectly* insulated (zero-heat-flow or adiabatic) surface. In practice, insulation is used to *reduce* heat flow and seldom can be regarded as perfect.



**Figure 1.4** Steady one-dimensional conduction across a plane wall, showing the application of the energy conservation principle to an elemental volume  $\Delta x$  thick.

$T_1 > T_2$ ,  $\dot{Q}$  is in the positive  $x$  direction.<sup>4</sup> The phenomenological law governing this heat flow is **Fourier's law of heat conduction**, which states that in a homogeneous substance, the local heat flux is proportional to the negative of the local temperature gradient:

$$\frac{\dot{Q}}{A} = q \quad \text{and} \quad q \propto -\frac{dT}{dx} \quad (1.7)$$

where  $q$  is the heat flux, or heat flow per unit area perpendicular to the flow direction [ $\text{W/m}^2$ ],  $T$  is the local temperature [K or  $^{\circ}\text{C}$ ], and  $x$  is the coordinate in the flow direction [m]. When  $dT/dx$  is negative, the minus sign in Eq. (1.7) gives a positive  $q$  in the positive  $x$  direction. Introducing a constant of proportionality  $k$ ,

$$q = -k \frac{dT}{dx} \quad (1.8)$$

where  $k$  is the **thermal conductivity** of the substance and, by inspection of the equation, must have units [ $\text{W/m K}$ ]. Notice that temperature can be given in kelvins or degrees Celsius in Eq. (1.8): the temperature gradient does not depend on which of these units is used since one kelvin equals one degree Celsius ( $1 \text{ K} = 1^{\circ}\text{C}$ ). Thus, the units of thermal conductivity could also be written [ $\text{W/m}^{\circ}\text{C}$ ], but this is not the recommended practice when using the SI system of units. The magnitude of the thermal conductivity  $k$  for a given substance very much depends on its microscopic structure and also tends to vary somewhat with temperature; Table 1.1 gives some selected values of  $k$ .

<sup>4</sup> Notice that this  $\dot{Q}$  is the heat flow in the  $x$  direction, whereas in the first law, Eqs. (1.1)–(1.4),  $\dot{Q} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$  is the net heat transfer into the whole system. In linking thermodynamics to heat transfer, some ambiguity in notation arises when common practice in both subjects is followed.

**Table 1.1** Selected values of thermal conductivity at 300 K ( $\sim 25^\circ\text{C}$ ).

Material	$k$ W/m K
Copper	386
Aluminum	204
Brass (70% Cu, 30% Zn)	111
Mild steel	64
Stainless steel, 18–8	15
Mercury	8.4
Concrete	1.4
Pyrex glass	1.09
Water	0.611
Neoprene rubber	0.19
Engine oil, SAE 50	0.145
White pine, perpendicular to grain	0.10
Polyvinyl chloride (PVC)	0.092
Freon 12	0.071
Cork	0.043
Fiberglass (medium density)	0.038
Polystyrene	0.028
Air	0.027

*Note:* Appendix A contains more comprehensive data.

Figure 1.4 shows an elemental volume  $\Delta V$  located between  $x$  and  $x + \Delta x$ ;  $\Delta V$  is a closed system, and the energy conservation principle in the form of Eq. (1.2) applies. If we consider a steady state, then temperatures are unchanging in time and  $dT/dt = 0$ ; also, if there is no heat generated within the volume,  $\dot{Q}_v = 0$ . Then Eq. (1.2) states that the net heat flow into the system is zero. Because the same amount of heat is flowing into  $\Delta V$  across the face at  $x$ , and out of  $\Delta V$  across the face at  $x + \Delta x$ ,

$$\dot{Q}|_x = \dot{Q}|_{x+\Delta x}$$

Since the rate of heat transfer is constant for all  $x$ , we simplify the notation by dropping the  $|_x$  and  $|_{x+\Delta x}$  subscripts (see the footnote on page 9), and write

$$\dot{Q} = \text{Constant}$$

But from Fourier's law, Eq. (1.8),

$$\dot{Q} = qA = -kA \frac{dT}{dx}$$

The variables are separable: rearranging and integrating across the wall,

$$\frac{\dot{Q}}{A} \int_0^L dx = - \int_{T_1}^{T_2} k dT$$

where  $\dot{Q}$  and  $A$  have been taken outside the integral signs since both are constants. If the small variation of  $k$  with temperature is ignored for the present we obtain

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) = \frac{T_1 - T_2}{L/kA} \quad (1.9)$$

Comparison of Eq. (1.9) with Ohm's law,  $I = E/R$ , suggests that  $\Delta T = T_1 - T_2$  can be viewed as a driving potential for flow of heat, analogous to voltage being the driving potential for current. Then  $R \equiv L/kA$  can be viewed as a **thermal resistance** analogous to electrical resistance.

If we have a composite wall of two slabs of material, as shown in Fig. 1.5, the heat flow through each layer is the same:

$$\dot{Q} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A}$$

Rearranging

$$\dot{Q} \left( \frac{L_A}{k_A A} \right) = T_1 - T_2$$

$$\dot{Q} \left( \frac{L_B}{k_B A} \right) = T_2 - T_3$$

Adding eliminates the interface temperature  $T_2$ :

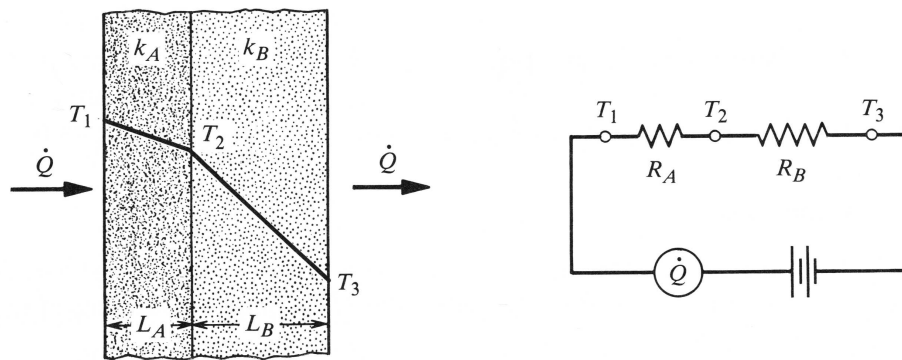
$$\dot{Q} \left( \frac{L_A}{k_A A} + \frac{L_B}{k_B A} \right) = T_1 - T_3$$

or

$$\dot{Q} = \frac{T_1 - T_3}{L_A/k_A A + L_B/k_B A} = \frac{\Delta T}{R_A + R_B} \quad (1.10a)$$

Using the electrical resistance analogy, we would view the problem as two resistances in series forming a **thermal circuit**, and immediately write

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \quad (1.10b)$$



**Figure 1.5** The temperature distribution for steady conduction across a composite plane wall and the corresponding thermal circuit.

**EXAMPLE 1.1** Heat Transfer through Insulation

A refrigerated container is in the form of a cube with 2 m sides and has 5 mm-thick aluminum walls insulated with a 10 cm layer of cork. During steady operation, the temperatures on the inner and outer surfaces of the container are measured to be  $-5^{\circ}\text{C}$  and  $20^{\circ}\text{C}$ , respectively. Determine the cooling load on the refrigerator.

**Solution**

**Given:** Aluminum container insulated with 10 cm—thick cork.

**Required:** Rate of heat gain.

**Assumptions:** 1. Steady state.  
2. One-dimensional heat conduction (ignore corner effects).

Equation (1.10) applies:

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \quad \text{where } R = \frac{L}{kA}$$

Let subscripts  $A$  and  $B$  denote the aluminum wall and cork insulation, respectively. Table 1.1 gives  $k_A = 204 \text{ W/m K}$ ,  $k_B = 0.043 \text{ W/m K}$ . We suspect that the thermal resistance of the aluminum wall is negligible, but we will calculate it anyway. For one side of area  $A = 4 \text{ m}^2$ , the thermal resistances are

$$R_A = \frac{L_A}{k_A A} = \frac{(0.005 \text{ m})}{(204 \text{ W/m K})(4 \text{ m}^2)} = 6.13 \times 10^{-6} \text{ K/W}$$

$$R_B = \frac{L_B}{k_B A} = \frac{(0.10 \text{ m})}{(0.043 \text{ W/m K})(4 \text{ m}^2)} = 0.581 \text{ K/W}$$

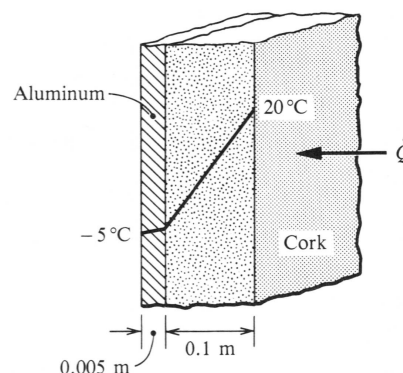
Since  $R_A$  is five orders of magnitude less than  $R_B$ , it can be ignored. The heat flow for a temperature difference of  $T_1 - T_2 = 20 - (-5) = 25 \text{ K}$ , is

$$\dot{Q} = \frac{\Delta T}{R_B} = \frac{25 \text{ K}}{0.581 \text{ K/W}} = 43.0 \text{ W}$$

For six sides, the total cooling load on the refrigerator is  $6.0 \times 43.0 = 258 \text{ W}$ .

**Comments**

1. In the future, when it is obvious that a resistance in a series network is negligible, it can be ignored from the outset (no effort should be expended to obtain data for its calculation).
2. The assumption of one-dimensional conduction is good because the 0.1 m insulation thickness is small compared to the 2 m-long sides of the cube.



3. Notice that the temperature difference  $T_1 - T_2$  is expressed in kelvins, even though  $T_1$  and  $T_2$  were given in degrees Celsius.
4. We have assumed perfect thermal contact between the aluminum and cork; that is, there is no thermal resistance associated with the interface between the two materials (see Section 2.2.2).

### 1.3.2 Thermal Radiation

All matter and space contains electromagnetic radiation. A particle, or *quantum*, of electromagnetic energy is a photon, and heat transfer by radiation can be viewed either in terms of electromagnetic waves or in terms of photons. The flux of radiant energy incident on a surface is its **irradiation**,  $G$  [ $\text{W/m}^2$ ]; the energy flux leaving a surface due to emission and reflection of electromagnetic radiation is its **radiosity**,  $J$  [ $\text{W/m}^2$ ]. A **black surface** (or **blackbody**) is defined as a surface that absorbs all incident radiation, reflecting none. As a consequence, all of the radiation leaving a black surface is emitted by the surface and is given by the **Stefan-Boltzmann law** as

$$J = E_b = \sigma T^4 \quad (1.11)$$

where  $E_b$  is the **blackbody emissive power**,  $T$  is absolute temperature [K], and  $\sigma$  is the Stefan-Boltzmann constant ( $\simeq 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ ). Table 1.2 shows how  $E_b = \sigma T^4$  increases rapidly with temperature.

**Table 1.2** Blackbody emissive power  $\sigma T^4$  at various temperatures.

Surface Temperature K	Blackbody Emissive Power $\text{W/m}^2$
300 (room temperature)	459
1000 (cherry-red hot)	56,700
3000 (lamp filament)	4,590,000
5760 (Sun temperature)	62,400,000

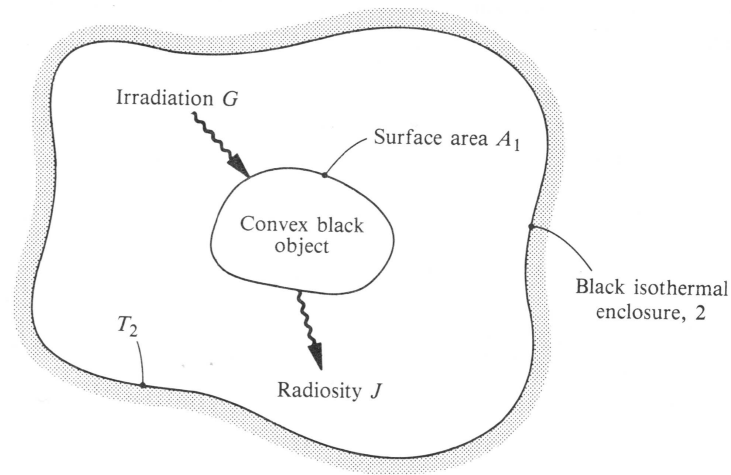
Figure 1.6 shows a convex black object of surface area  $A_1$  in an evacuated black isothermal enclosure at temperature  $T_2$ . At equilibrium, the object is also at temperature  $T_2$ , and the radiant energy incident on the object must equal the radiant energy leaving from the object:

$$G_1 A_1 = J_1 A_1 = \sigma T_2^4 A_1$$

Hence

$$G_1 = \sigma T_2^4 \quad (1.12)$$

and is uniform over the area. If the temperature of the object is now raised to  $T_1$ , its radiosity becomes  $\sigma T_1^4$  while its irradiation remains  $\sigma T_2^4$  (because the enclosure reflects no radiation). Then the net radiant heat flux through the surface,  $q_1$ , is the



**Figure 1.6** A convex black object (surface 1) in a black isothermal enclosure (surface 2).

radiosity minus the irradiation:

$$q_1 = J_1 - G_1 \quad (1.13)$$

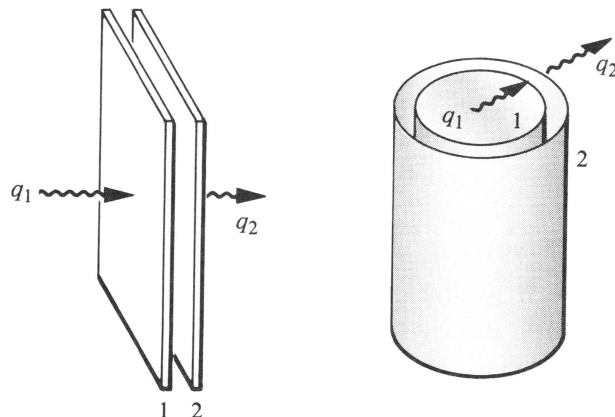
or

$$q_1 = \sigma T_1^4 - \sigma T_2^4 \quad (1.14)$$

where the sign convention is such that a net flux away from the surface is positive. Equation (1.14) is also valid for two large black surfaces facing each other, as shown in Fig. 1.7.

The blackbody is an ideal surface. Real surfaces absorb less radiation than do black surfaces. The fraction of incident radiation absorbed is called the absorptance (or absorptivity),  $\alpha$ . A widely used model of a real surface is the **gray surface**, which is defined as a surface for which  $\alpha$  is a constant, irrespective of the nature of the incident radiation. The fraction of incident radiation reflected is the **reflectance** (or reflectivity),  $\rho$ . If the object is opaque, that is, not transparent to electromagnetic radiation, then

$$\rho = 1 - \alpha \quad (1.15)$$



**Figure 1.7** Examples of two large surfaces facing each other.

**Table 1.3** Selected approximate values of emittance,  $\varepsilon$  (total hemispherical values at normal temperatures).

Surface	Emittance, $\varepsilon$
Aluminum alloy, unoxidized	0.035
Black anodized aluminum	0.80
Chromium plating	0.16
Stainless steel, type 312, lightly oxidized	0.30
Inconel X, oxidized	0.72
Black enamel paint	0.78
White acrylic paint	0.90
Asphalt	0.88
Concrete	0.90
Soil	0.94
Pyrex glass	0.80

*Note:* More comprehensive data are given in Appendix A. Emittance is very dependent on surface finish; thus, values obtained from various sources may differ significantly.

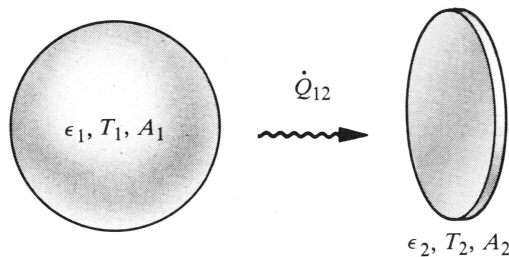
Real surfaces also emit less radiation than do black surfaces. The fraction of the blackbody emissive power  $\sigma T^4$  emitted is called the **emittance** (or emissivity),  $\varepsilon$ .<sup>5</sup> A gray surface also has a constant value of  $\varepsilon$ , independent of its temperature, and, as will be shown in Chapter 6, the emittance and absorptance of a gray surface are equal:

$$\varepsilon = \alpha \text{ (gray surface)} \quad (1.16)$$

Table 1.3 shows some typical values of  $\varepsilon$  at normal temperatures. Bright metal surfaces tend to have low values, whereas oxidized or painted surfaces tend to have high values. Values of  $\alpha$  and  $\rho$  can also be obtained from Table 1.3 by using Eqs. (1.15) and (1.16).

If heat is transferred by radiation between two gray surfaces of finite size, as shown in Fig. 1.8, the rate of heat flow will depend on temperatures  $T_1$  and  $T_2$  and emittances  $\varepsilon_1$  and  $\varepsilon_2$ , as well as the geometry. Clearly, some of the radiation leaving surface 1 will not be intercepted by surface 2, and vice versa. Determining the rate of heat flow is usually quite difficult. In general, we may write

$$\dot{Q}_{12} = A_1 \mathcal{F}_{12} (\sigma T_1^4 - \sigma T_2^4) \quad (1.17)$$

**Figure 1.8** Radiation heat transfer between two finite gray surfaces.

<sup>5</sup> Both the endings *-ance* and *-ivity* are commonly used for radiation properties. In this text, *-ance* will be used for surface radiation properties. In Chapter 6, *-ivity* will be used for gas radiation properties.

where  $\dot{Q}_{12}$  is the net radiant energy interchange (heat transfer) from surface 1 to surface 2, and  $\mathcal{F}_{12}$  is a **transfer factor**, which depends on emittances and geometry. For the special case of surface 1 surrounded by surface 2, where either area  $A_1$  is small compared to area  $A_2$ , or surface 2 is nearly black,  $\mathcal{F}_{12} \simeq \varepsilon_1$  and Eq. (1.17) becomes

$$\dot{Q}_{12} = \varepsilon_1 A_1 (\sigma T_1^4 - \sigma T_2^4) \quad (1.18)$$

Equation (1.18) will be derived in Chapter 6. It is an important result and is often used for quick engineering estimates.

The  $T^4$  dependence of radiant heat transfer complicates engineering calculations. When  $T_1$  and  $T_2$  are not too different, it is convenient to linearize Eq. (1.18) by factoring the term  $(\sigma T_1^4 - \sigma T_2^4)$  to obtain

$$\begin{aligned} \dot{Q}_{12} &= \varepsilon_1 A_1 \sigma (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2) \\ &\simeq \varepsilon_1 A_1 \sigma (4T_m^3)(T_1 - T_2) \end{aligned}$$

for  $T_1 \simeq T_2$ , where  $T_m$  is the mean of  $T_1$  and  $T_2$ . This result can be written more concisely as

$$\dot{Q}_{12} \simeq A_1 h_r (T_1 - T_2) \quad (1.19)$$

where  $h_r = 4\varepsilon_1 \sigma T_m^3$  is called the **radiation heat transfer coefficient** [ $\text{W/m}^2 \text{K}$ ]. At  $25^\circ\text{C}$  ( $= 298 \text{ K}$ ),

$$h_r = (4)\varepsilon_1 (5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4) (298 \text{ K})^3$$

or

$$h_r \simeq 6\varepsilon_1 \text{ W/m}^2 \text{K}$$

This result can be easily remembered: The radiation heat transfer coefficient at room temperature is about six times the surface emittance. For  $T_1 = 320 \text{ K}$  and  $T_2 = 300 \text{ K}$ , the error incurred in using the approximation of Eq. (1.19) is only 0.1%; for  $T_1 = 400 \text{ K}$  and  $T_2 = 300 \text{ K}$ , the error is 2%.

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### EXAMPLE 1.2 Heat Loss from a Transistor

An electronic package for an experiment in outer space contains a transistor capsule, which is approximately spherical in shape with a 2 cm diameter. It is contained in an evacuated case with nearly black walls at  $30^\circ\text{C}$ . The only significant path for heat loss from the capsule is radiation to the case walls. If the transistor dissipates 300 mW, what will the capsule temperature be if it is (i) bright aluminum and (ii) black anodized aluminum?

#### Solution

**Given:** 2 cm-diameter transistor capsule dissipating 300 mW.

**Required:** Capsule temperature for (i) bright aluminum and (ii) black anodized aluminum.

**Assumptions:** Model as a small gray body in large, nearly black surroundings.

Equation (1.18) is applicable with

$$\dot{Q}_{12} = 300 \text{ mW}$$

$$T_2 = 30^\circ\text{C} = 303 \text{ K}$$

and  $T_1$  is the unknown.

$$\dot{Q}_{12} = \varepsilon_1 A_1 (\sigma T_1^4 - \sigma T_2^4)$$

$$0.3 \text{ W} = (\varepsilon_1)(\pi)(0.02 \text{ m})^2 [\sigma T_1^4 - (5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(303 \text{ K})^4]$$

Solving,

$$\sigma T_1^4 = 478 + \frac{239}{\varepsilon_1}$$

(i) For bright aluminum ( $\varepsilon = 0.035$  from Table 1.3),

$$\sigma T_1^4 = 478 + 6828 = 7306 \text{ W/m}^2$$

$$T_1 = 599 \text{ K } (326^\circ\text{C})$$

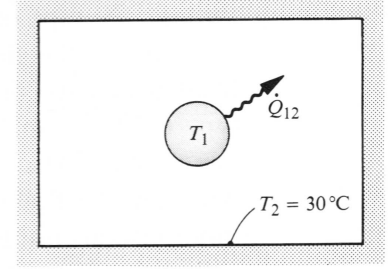
(ii) For black anodized aluminum ( $\varepsilon = 0.80$  from Table 1.3),

$$\sigma T_1^4 = 478 + 298 = 776 \text{ W/m}^2$$

$$T_1 = 342 \text{ K } (69^\circ\text{C})$$

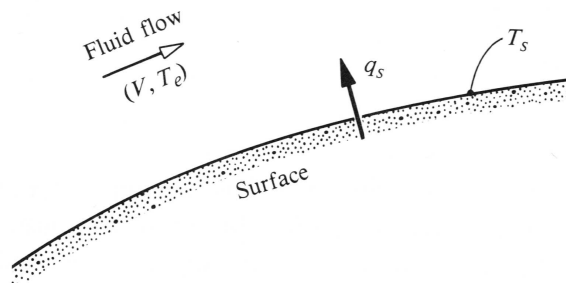
### Comments

1. The anodized aluminum gives a satisfactory operating temperature, but a bright aluminum capsule could not be used since  $326^\circ\text{C}$  is far in excess of allowable operating temperatures for semiconductor devices.
2. Note the use of kelvins for temperature in this radiation heat transfer calculation.



### 1.3.3 Heat Convection

As already explained, *convection* or *convective heat transfer* is the term used to describe heat transfer from a surface to a moving fluid, as shown in Fig. 1.9. The surface may be the inside of a pipe, the skin of a hypersonic aircraft, or a water-air interface in a cooling tower. The flow may be *forced*, as in the case of a liquid pumped through



**Figure 1.9** Schematic of convective heat transfer to a fluid at temperature  $T_e$  flowing at velocity  $V$  past a surface at temperature  $T_s$ .

the pipe or air on the flight vehicle propelled through the atmosphere. On the other hand, the flow could be *natural* (or *free*), driven by buoyancy forces arising from a density difference, as in the case of a natural-draft cooling tower. Either type of flow can be *internal*, such as the pipe flow, or *external*, such as flow over the vehicle. Also, both forced and natural flows can be either *laminar* or *turbulent*, with laminar flows being predominant at lower velocities, for smaller sizes, and for more viscous fluids. Flow in a pipe may become turbulent when the dimensionless group called the **Reynolds number**,  $Re_D = VD/\nu$ , exceeds about 2300, where  $V$  is the velocity [m/s],  $D$  is the pipe diameter [m], and  $\nu$  is the kinematic viscosity of the fluid [m<sup>2</sup>/s]. Heat transfer rates tend to be much higher in turbulent flows than in laminar flows, owing to the vigorous mixing of the fluid. Figure 1.10 shows some commonly encountered flows.

The rate of heat transfer by convection is usually a complicated function of surface geometry and temperature, the fluid temperature and velocity, and fluid thermo-physical properties. In an external forced flow, the rate of heat transfer is approximately proportional to the difference between the surface temperature  $T_s$  and the temperature of the free stream fluid  $T_e$ . The constant of proportionality is called the **convective heat transfer coefficient**  $h_c$ :

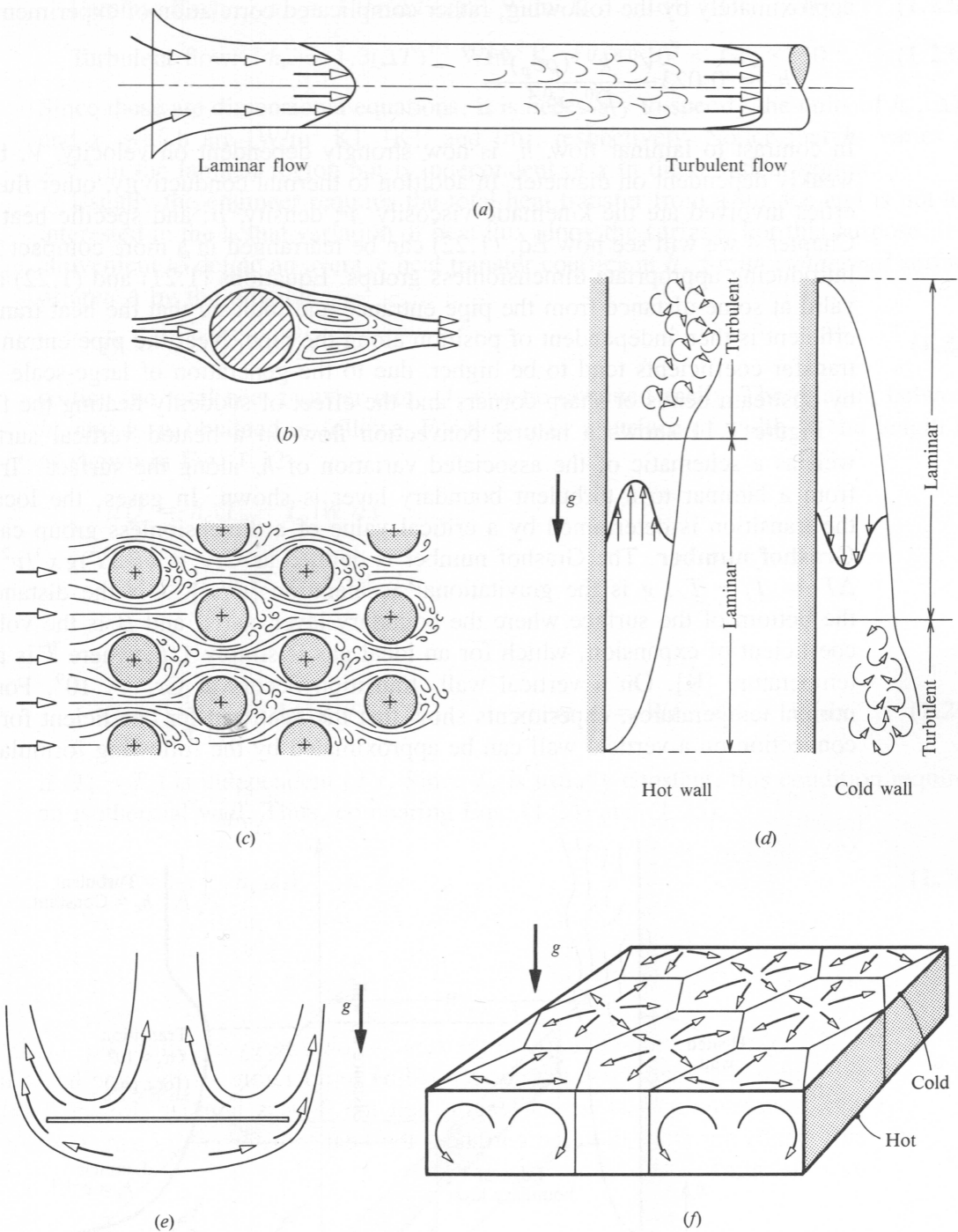
$$q_s = h_c \Delta T \quad (1.20)$$

where  $\Delta T = T_s - T_e$ ,  $q_s$  is the heat flux from the surface into the fluid [W/m<sup>2</sup>], and  $h_c$  has units [W/m<sup>2</sup> K]. Equation (1.20) is often called *Newton's law of cooling* but is a definition of  $h_c$  rather than a true physical law. For natural convection, the situation is more complicated. If the flow is laminar,  $q_s$  varies as  $\Delta T^{5/4}$ ; if the flow is turbulent, it varies as  $\Delta T^{4/3}$ . However, we still find it convenient to define a heat transfer coefficient by Eq. (1.20); then  $h_c$  varies as  $\Delta T^{1/4}$  for laminar flows and as  $\Delta T^{1/3}$  for turbulent ones.

An important practical problem is convective heat transfer to a fluid flowing in a tube, as may be found in heat exchangers for heating or cooling liquids, in condensers, and in various kinds of boilers. In using Eq. (1.20) for internal flows,  $\Delta T = T_s - T_b$ , where  $T_b$  is a properly averaged fluid temperature called the **bulk temperature** or mixed mean temperature and is defined in Chapter 4. Here it is sufficient to note that enthalpy in the steady-flow energy equation, Eq. (1.4), is also the bulk value, and  $T_b$  is the corresponding temperature. If the pipe has a uniform wall temperature  $T_s$  along its length, and the flow is laminar ( $Re_D \lesssim 2300$ ), then sufficiently far from the pipe entrance, the heat transfer coefficient is given by the exact relation

$$h_c = 3.66 \frac{k}{D} \quad (1.21)$$

where  $k$  is the fluid thermal conductivity and  $D$  is the pipe diameter. Notice that the heat transfer coefficient is directly proportional to thermal conductivity, inversely proportional to pipe diameter, and—perhaps surprisingly— independent of flow velocity. On the other hand, for fully turbulent flow ( $Re_D \gtrsim 10,000$ ),  $h_c$  is given



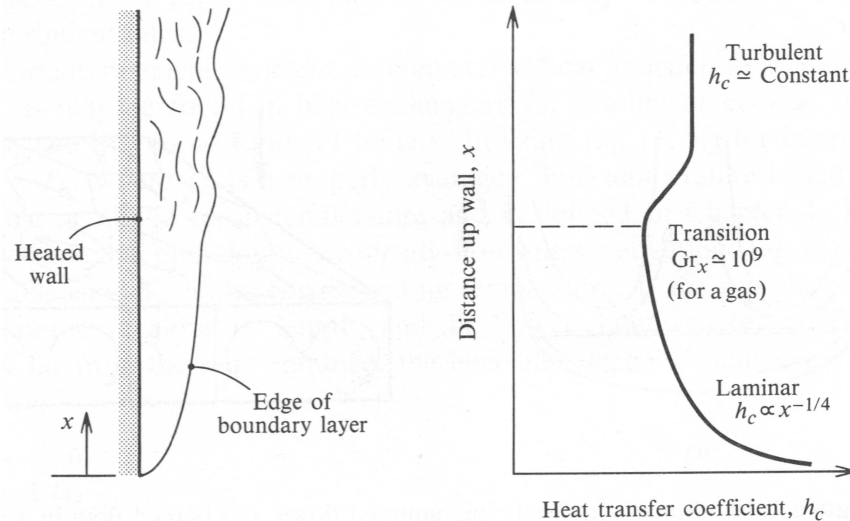
**Figure 1.10** Some commonly encountered flows, (a) Forced flow in a pipe,  $Re_D \simeq 50,000$ . The flow is initially laminar because of the “bell-mouth” entrance but becomes turbulent downstream, (b) Laminar forced flow over a cylinder,  $Re_D \simeq 25$ . (c) Forced flow through a tube bank as found in a shell-and-tube heat exchanger, (d) Laminar and turbulent natural convection boundary layers on vertical walls, (e) Laminar natural convection about a heated horizontal plate, (f) Cellular natural convection in a horizontal enclosed fluid layer.

approximately by the following, rather complicated correlation of experimental data:

$$h_c = 0.023 \frac{V^{0.8} k^{0.6} (\rho c_p)^{0.4}}{D^{0.2} \nu^{0.4}} \quad (1.22)$$

In contrast to laminar flow,  $h_c$  is now strongly dependent on velocity,  $V$ , but only weakly dependent on diameter. In addition to thermal conductivity, other fluid properties involved are the kinematic viscosity,  $\nu$ ; density,  $\rho$ ; and constant-pressure specific heat,  $c_p$ . In Chapter 4 we will see how Eq. (1.22) can be rearranged in a more compact form by introducing appropriate dimensionless groups. Equations (1.21) and (1.22) are only valid at some distance from the pipe entrance and indicate that the heat transfer coefficient is then independent of position along the pipe. Near the pipe entrance, heat transfer coefficients tend to be higher, due to the generation of large-scale vortices by upstream bends or sharp corners and the effect of suddenly heating the fluid.

Figure 1.11 shows a natural convection flow on a heated vertical surface, as well as a schematic of the associated variation of  $h_c$  along the surface. Transition from a laminar to a turbulent boundary layer is shown. In gases, the location of the transition is determined by a critical value of a dimensionless group called the **Grashof number**. The Grashof number is defined as  $Gr_x = (\beta \Delta T) g x^3 / \nu^2$ , where  $\Delta T = T_s - T_e$ ,  $g$  is the gravitational acceleration [ $\text{m/s}^2$ ],  $x$  is the distance from the bottom of the surface where the boundary layer starts, and  $\beta$  is the volumetric coefficient of expansion, which for an ideal gas is simply  $1/T$ , where  $T$  is absolute temperature [K]. On a vertical wall, transition occurs at  $Gr_x \approx 10^9$ . For air, at normal temperatures, experiments show that the heat transfer coefficient for natural convection on a vertical wall can be approximated by the following formulas:



**Figure 1.11** A natural-convection boundary layer on a vertical wall, showing the variation of local heat transfer coefficient. For gases, transition from a laminar to turbulent flow occurs at a Grashof number of approximately  $10^9$ ; hence  $x_{tr} \simeq [10^9 \nu^2 / \beta \Delta T g]^{1/3}$ .

$$\text{Laminar flow: } h_c = 1.07(\Delta T/x)^{1/4} \text{ W/m}^2\text{K} \quad 10^4 < \text{Gr}_x < 10^9 \quad (1.23a)$$

$$\text{Turbulent flow: } h_c = 1.3(\Delta T)^{1/3} \text{ W/m}^2\text{K} \quad 10^9 < \text{Gr}_x < 10^{12} \quad (1.23b)$$

Since these are dimensional equations, it is necessary to specify the units of  $h_c$ ,  $\Delta T$ , and  $x$ , which are  $[\text{W/m}^2 \text{K}]$ ,  $[\text{K}]$ , and  $[\text{m}]$ , respectively. Notice that  $h_c$  varies as  $x^{-1/4}$  in the laminar region but is independent of  $x$  in the turbulent region.

Usually the engineer requires the total heat transfer from a surface and is not too interested in the actual variation of heat flux along the surface. For this purpose, it is convenient to define an average heat transfer coefficient  $\bar{h}_c$  for an *isothermal* surface of area  $A$  by the relation

$$\dot{Q} = \bar{h}_c A (T_s - T_e) \quad (1.24)$$

so that the total heat transfer rate,  $\dot{Q}$ , can be obtained easily. The relation between  $\bar{h}_c$  and  $h_c$  is obtained as follows: For flow over a surface of width  $W$  and length  $L$ , as shown in Fig. 1.12,

$$d\dot{Q} = h_c (T_s - T_e) W dx$$

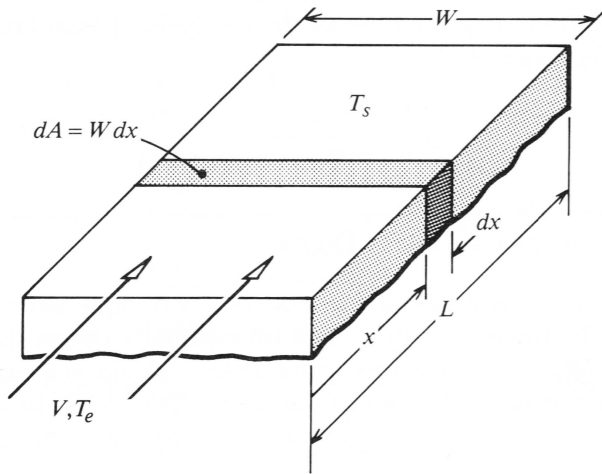
$$\dot{Q} = \int_0^L h_c (T_s - T_e) W dx$$

or

$$\dot{Q} = \left( \frac{1}{A} \int_0^A h_c dA \right) A (T_s - T_e), \quad \text{where } A = WL, dA = W dx \quad (1.25)$$

if  $(T_s - T_e)$  is independent of  $x$ . Since  $T_e$  is usually constant, this condition requires an isothermal wall. Thus, comparing Eqs. (1.24) and (1.25),

$$\bar{h}_c = \frac{1}{A} \int_0^A h_c dA \quad (1.26)$$



**Figure 1.12** An isothermal surface used to define the average convective heat transfer coefficient  $\bar{h}_c$ .

**Table 1.4** Orders of magnitude of average convective heat transfer coefficients.

Flow and Fluid	$\bar{h}_c$ W/m <sup>2</sup> K
Free convection, air	3–25
Free convection, water	15–1000
Forced convection, air	10–200
Forced convection, water	50–10,000
Forced convection, liquid sodium	10,000–100,000
Condensing steam	5000–50,000
Boiling water	3000–100,000

The surface may not be isothermal; for example, the surface may be electrically heated to give a uniform flux  $q_s$  along the surface. In this case, defining an average heat transfer coefficient is more difficult and will be dealt with in Chapter 4. Table 1.4 gives some order-of-magnitude values of average heat transfer coefficients for various situations. In general, high heat transfer coefficients are associated with high fluid thermal conductivities, high flow velocities, and small surfaces. The high heat transfer coefficients shown for boiling water and condensing steam are due to another cause: as we will see in Chapter 7, a large enthalpy of phase change (latent heat) is a contributing factor.

The complexity of most situations involving convective heat transfer precludes exact analysis, and *correlations* of experimental data must be used in engineering practice. For a particular situation, a number of correlations from various sources might be available, for example, from research laboratories in different countries. Also, as time goes by, older correlations may be superseded by newer correlations based on more accurate or more extensive experimental data. Heat transfer coefficients calculated from various available correlations usually do not differ by more than about 20%, but in more complex situations, much larger discrepancies may be encountered. Such is the nature of engineering calculations of convective heat transfer, in contrast to the more exact nature of the analysis of heat conduction or of elementary mechanics, for example.

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### EXAMPLE 1.3 Heat Loss through Glass Doors

The living room of a ski chalet has a pair of glass doors 2.3 m high and 4.0 m wide. On a cold morning, the air in the room is at 10°C, and frost partially covers the inner surface of the glass. Estimate the convective heat loss to the doors. Would you expect to see the frost form initially near the top or the bottom of the doors? Take  $\nu = 14 \times 10^{-6}$  m<sup>2</sup>/s for the air.

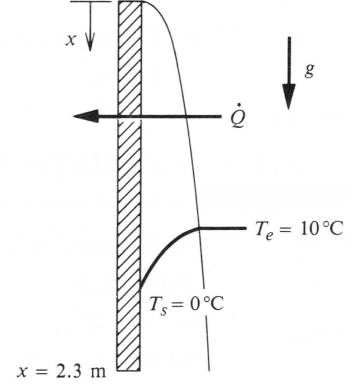
**Solution**

**Given:** Glass doors, width  $W = 4$  m, height  $L = 2.3$  m.

**Required:** Estimate of convective heat loss to the doors.

**Assumptions:** 1. Inner surface isothermal at  $T_s \simeq 0^\circ\text{C}$ .  
2. The laminar to turbulent flow transition occurs at  $Gr_x \simeq 10^9$ .

Equation (1.24) will be used to estimate the heat loss. The inner surface will be at approximately  $0^\circ\text{C}$  since it is only partially covered with frost. If it were warmer, frost couldn't form; and if it were much colder, frost would cover the glass completely. There is a natural convection flow down the door since  $T_e = 10^\circ\text{C}$  is greater than  $T_s = 0^\circ\text{C}$ . Transition from a laminar boundary layer to a turbulent boundary layer occurs when the Grashof number is about  $10^9$ . For transition at  $x = x_{tr}$ ,



$$Gr = 10^9 = \frac{(\beta \Delta T) g x_{tr}^3}{\nu^2}; \quad \beta = 1/T \text{ for an ideal gas}$$

$$x_{tr} = \left[ \frac{10^9 \nu^2}{(\Delta T / T) g} \right]^{1/3} = \left[ \frac{(10^9)(14 \times 10^{-6} \text{ m}^2/\text{s})^2}{(10/278)(9.81 \text{ m/s}^2)} \right]^{1/3} = 0.82 \text{ m}$$

where the average of  $T_s$  and  $T_e$  has been used to evaluate  $\beta$ . The transition is seen to take place about one third of the way down the door.

We find the average heat transfer coefficient,  $\bar{h}_c$ , by substituting Eqs. (1.23 a,b) in Eq. (1.26):

$$\begin{aligned} \bar{h}_c &= \frac{1}{A} \int_0^A h_c dA; \quad A = WL, \quad dA = W dx \\ &= \frac{1}{L} \int_0^L h_c dx \\ &= \frac{1}{L} \left[ \int_0^{x_{tr}} 1.07(\Delta T/x)^{1/4} dx + \int_{x_{tr}}^L 1.3(\Delta T)^{1/3} dx \right] \\ &= (1/L) [(1.07)(4/3)\Delta T^{1/4} x_{tr}^{3/4} + (1.3)(\Delta T)^{1/3} (L - x_{tr})] \\ &= (1/2.3) [(1.07)(4/3)(10)^{1/4} (0.82)^{3/4} + (1.3)(10)^{1/3} (2.3 - 0.82)] \\ &= (1/2.3) [2.19 + 4.15] \\ &= 2.75 \text{ W/m}^2\text{K} \end{aligned}$$

Then, from Eq. (1.24), the total heat loss to the door is

$$\dot{Q} = \bar{h}_c A \Delta T = (2.75 \text{ W/m}^2 \text{ K})(2.3 \times 4.0 \text{ m}^2)(10 \text{ K}) = 253 \text{ W}$$

### Comments

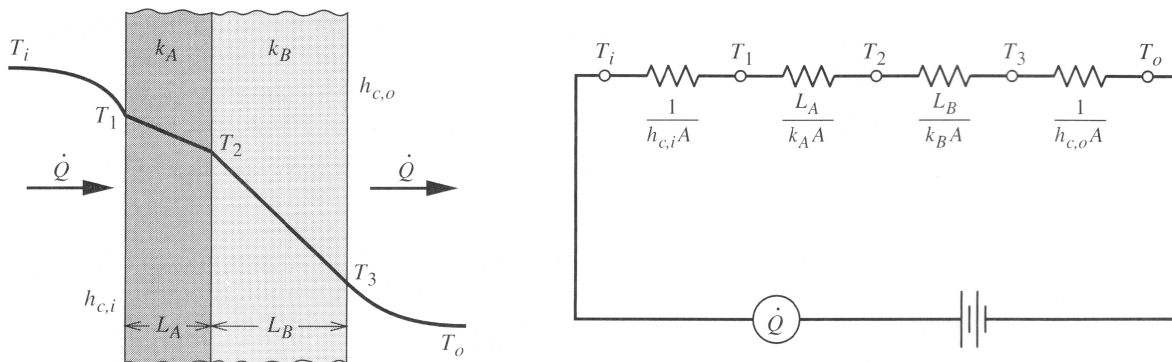
1. The local heat transfer coefficient is larger near the top of the door, so that the relatively warm room air will tend to cause the glass there to be at a higher temperature than further down the door. Thus, frost should initially form near the bottom of the door.
2. In addition, interior surfaces in the room will lose heat by radiation through the glass doors.

## 1.4 COMBINED MODES OF HEAT TRANSFER

Heat transfer problems encountered by the design engineer almost always involve more than one mode of heat transfer occurring simultaneously. For example, consider the nighttime heat loss through the roof of the house shown in Fig. 1.3. Heat is transferred to the ceiling by convection from the warm room air, and by radiation from the walls, furniture, and occupants. Heat transfer across the ceiling and its insulation is by conduction, across the attic crawlspace by convection and radiation, and across the roof tile by conduction. Finally, the heat is transferred by convection to the cold ambient air, and by radiation to the nighttime sky. To consider realistic engineering problems, it is necessary at the outset to develop the theory required to handle *combined modes* of heat transfer.

### 1.4.1 Thermal Circuits

The electrical circuit analogy for conduction through a composite wall was introduced in Section 1.3.1. We now extend this concept to include convection and radiation as well. Figure 1.13 shows a two-layer composite wall of cross-sectional area  $A$  with the layers  $A$  and  $B$  having thickness and conductivity  $L_A, k_A$  and  $L_B, k_B$ , respectively. Heat is transferred from a hot fluid at temperature  $T_i$  to the inside of the wall with a convective heat transfer coefficient  $h_{c,i}$ , and away from the outside of the wall to a cold fluid at temperature  $T_o$  with heat transfer coefficient  $h_{c,o}$ .



**Figure 1.13** The temperature distribution for steady heat transfer across a composite plane wall, and the corresponding thermal circuit.

Newton's law of cooling, Eq. (1.20), can be rewritten as

$$\dot{Q} = \frac{\Delta T}{1/h_c A} \quad (1.27)$$

with  $1/h_c A$  identified as a convective thermal resistance. At steady state, the heat flow through the wall is constant. Referring to Fig. 1.13 for the intermediate temperatures,

$$\dot{Q} = \frac{T_i - T_1}{1/h_{c,i}A} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A} = \frac{T_3 - T_o}{1/h_{c,o}A} \quad (1.28)$$

Equation (1.28) is the basis of the thermal circuit shown in Fig. 1.13. The total resistance is the sum of four resistances in series. If we define the **overall heat transfer coefficient**  $U$  by the relation

$$\dot{Q} = UA(T_i - T_o) \quad (1.29)$$

then  $1/UA$  is an overall resistance given by

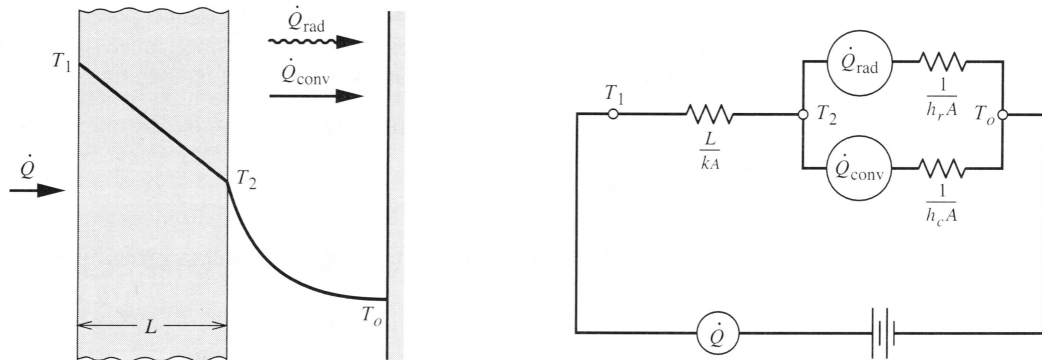
$$\frac{1}{UA} = \frac{1}{h_{c,i}A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_{c,o}A} \quad (1.30a)$$

or, since the cross-sectional area  $A$  is constant for a plane wall,

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{1}{h_{c,o}} \quad (1.30b)$$

Equation (1.29) is simple and convenient for use in engineering calculations. Typical values of  $U$  [ $\text{W/m}^2 \text{ K}$ ] vary over a wide range for different types of walls and convective flows.

Figure 1.14 shows a wall whose outer surface loses heat by both convection and radiation. For simplicity, assume that the fluid is at the same temperature as the surrounding surfaces,  $T_o$ . Using the approximate linearized Eq. (1.19),



**Figure 1.14** A wall that loses heat by both conduction and radiation; the thermal circuit shows resistances in parallel.

$$\dot{Q}_{\text{rad}} = \frac{\Delta T}{1/h_r A} \quad (1.31)$$

with  $1/h_r A$  identified as a radiative thermal resistance. We now have two resistances in parallel, as shown in Fig. 1.14. The sum of the resistances is

$$\sum R = \frac{L}{kA} + \frac{1}{h_c A + h_r A}$$

so

$$\frac{1}{UA} = \frac{L}{kA} + \frac{1}{(h_c + h_r)A} \quad (1.32)$$

so that the convective and radiative heat transfer coefficients can simply be added. However, often the fluid and surrounding temperatures are not the same, or the simple linearized representation of radiative transfer [Eq. (1.19)] is invalid, so the thermal circuit is then more complex. When appropriate, we will write  $h = h_c + h_r$  to account for combined convection and radiation.<sup>6</sup>

#### EXAMPLE 1.4 Heat Loss through a Composite Wall

The walls of a sparsely furnished single-room cabin in a forest consist of two layers of pine wood, each 2 cm thick, sandwiching 5 cm of fiberglass insulation. The cabin interior is maintained at 20°C when the ambient air temperature is 2°C. If the interior and exterior convective heat transfer coefficients are 3 and 6 W/m<sup>2</sup> K, respectively, and the exterior surface is finished with a white acrylic paint, estimate the heat flux through the wall.

#### Solution

**Given:** Pine wood cabin wall insulated with 5 cm of fiberglass.

**Required:** Estimate of heat loss through wall.

**Assumptions:** 1. Forest trees and shrubs are at the ambient air temperature,  $T_e = 2^\circ\text{C}$ .  
2. Radiation transfer inside cabin is negligible since inner surfaces of walls, roof, and floor are at approximately the same temperature.

From Eq. (1.29), the heat flux through the wall is

$$q = \frac{\dot{Q}}{A} = U(T_i - T_o)$$

From Eqs. (1.30) and (1.32), the overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{(h_{c,o} + h_{r,o})}$$

<sup>6</sup> Notice that the notation used for this combined heat transfer coefficient,  $h$ , is the same as that used for enthalpy. The student must be careful not to confuse these two quantities. Other notation is also in common use, for example,  $\alpha$  for the heat transfer coefficient and  $i$  for enthalpy.

The thermal conductivities of pine wood, perpendicular to the grain, and of fiberglass are given in Table 1.1 as 0.10 and 0.038 W/m K, respectively. The exterior radiation heat transfer coefficient is given by Eq. (1.19) as

$$h_{r,o} = 4\epsilon\sigma T_m^3$$

where  $\epsilon = 0.9$  for white acrylic paint, from Table 1.3, and  $T_m \simeq 2^\circ\text{C} = 275\text{ K}$  (since we expect the exterior resistance to be small). Thus,

$$h_{r,o} = 4(0.9)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(275\text{K})^3$$

$$= 4.2 \text{ W/m}^2 \text{ K}$$

$$\frac{1}{U} = \frac{1}{3} + \frac{0.02}{0.10} + \frac{0.05}{0.038} + \frac{0.02}{0.10} + \frac{1}{6 + 4.2}$$

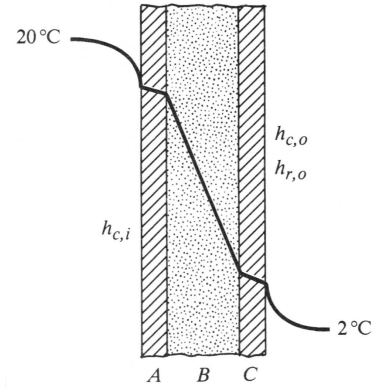
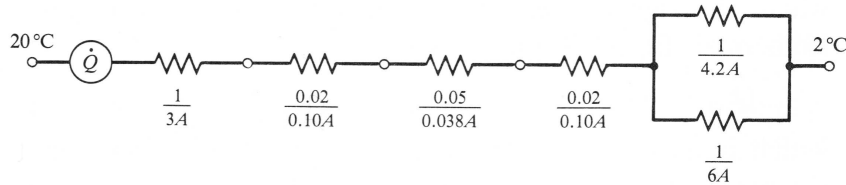
$$= 0.333 + 0.200 + 1.316 + 0.200 + 0.098$$

$$= 2.15 \text{ (W/m}^2 \text{ K)}^{-1}$$

$$U = 0.466 \text{ W/m}^2 \text{ K}$$

Then the heat flux  $q = U(T_i - T_o) = 0.466(20 - 2) = 8.38 \text{ W/m}^2$ .

The thermal circuit is shown below.

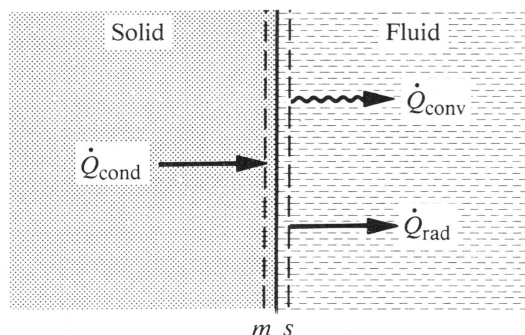


### Comments

1. The outside resistance is seen to be  $0.098/2.15 \simeq 5\%$  of the total resistance; hence, the outside wall of the cabin is only about 1 K above the ambient air, and our assumption of  $T_m = 275\text{ K}$  for the evaluation of  $h_{r,o}$  is adequate.
2. The dominant resistance is that of the fiberglass insulation; therefore, an accurate calculation of  $q$  depends mainly on having accurate values for the fiberglass thickness and thermal conductivity. Poor data or poor assumptions for the other resistances have little impact on the result.

### 1.4.2 Surface Energy Balances

Section 1.4.1 assumed that the energy flow  $\dot{Q}$  across the wall surfaces is continuous. In fact, we used a procedure commonly called a *surface energy balance*, which is used in various ways. Some examples follow. Figure 1.15 shows an opaque solid that is losing heat by convection and radiation to its surroundings. Two imaginary surfaces are located on each side of the real solid-fluid interface: an  $s$ -surface in



**Figure 1.15** Schematic of a surface energy balance, showing the  $m$ - and  $s$ -surface in the solid and fluid, respectively.

the fluid just adjacent to the interface, and an  $m$ -surface in the solid located such that all radiation is emitted or absorbed between the  $m$ -surface and the interface. Thus, energy is transferred across the  $m$ -surface by conduction only. (The choice of  $s$  and  $m$  to designate these surfaces follows an established practice. In particular, the use of the  $s$  prefix is consistent with the use of the subscript  $s$  to denote a surface temperature  $T_s$ , in convection analysis.) The first law as applied to the closed system located between  $m$ - and  $s$ -surfaces requires that  $\sum \dot{Q} = 0$ ; thus,

$$\dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{rad}} = 0 \quad (1.33)$$

or, for a unit area,

$$q_{\text{cond}} - q_{\text{conv}} - q_{\text{rad}} = 0 \quad (1.34)$$

where the sign convention for the fluxes is shown in Fig. 1.15. If the solid is isothermal, Eq. (1.33) reduces to

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0 \quad (1.35)$$

which is a simple energy balance on the solid. Notice that these surface energy balances remain valid for unsteady conditions, in which temperatures change with time, provided the mass contained between the  $s$ - and  $m$ -surfaces is negligible and cannot store energy.

---

### EXAMPLE 1.5 Air Temperature Measurement

A machine operator in a workshop complains that the air-heating system is not keeping the air at the required minimum temperature of 20°C. To support his claim, he shows that a mercury-in-glass thermometer suspended from a roof truss reads only 17°C. The roof and walls of the workshop are made of corrugated iron and are not insulated; when the thermometer is held against the wall, it reads only 5°C. If the average convective heat transfer coefficient for the suspended thermometer is estimated to be 10 W/m<sup>2</sup> K, what is the true air temperature?

#### Solution

**Given:** Thermometer reading a temperature of 17°C.

**Required:** True air temperature.

**Assumptions:** Thermometer can be modeled as a small gray body in large, nearly black surroundings at 5°C.

Let  $T_t$  be the thermometer reading,  $T_e$  the air temperature, and  $T_w$  the wall temperature. Equation (1.35) applies,

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0$$

since at steady state there is no conduction within the thermometer. Substituting from Eqs. (1.24) and (1.18),

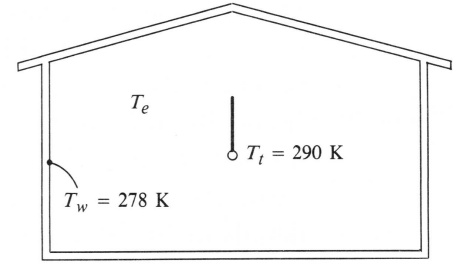
$$\bar{h}_c A (T_t - T_e) + \varepsilon \sigma A (T_t^4 - T_w^4) = 0$$

From Table 1.3,  $\varepsilon = 0.8$  for pyrex glass. Canceling  $A$ ,

$$10(290 - T_e) + (0.8)(5.67)(2.90^4 - 2.78^4) = 0$$

Solving,

$$T_e = 295 \text{ K} \simeq 22^\circ\text{C}$$



### Comments

1. Since  $T_e > 20^\circ\text{C}$ , the air-heating system appears to be working satisfactorily.
2. Our model assumes that the thermometer is completely surrounded by a surface at  $5^\circ\text{C}$ : actually, the thermometer also receives radiation from machines, workers, and other sources at temperatures higher than  $5^\circ\text{C}$ , so that our calculated value of  $T_e = 22^\circ\text{C}$  is somewhat high.

## 1.5 TRANSIENT THERMAL RESPONSE

The heat transfer problems described in Examples 1.1 through 1.5 were *steady-state* problems; that is, temperatures were not changing in time. In Example 1.2, the transistor temperature was steady with the resistance ( $I^2R$ ) heating balanced by the radiation heat loss. *Unsteady-state* or *transient* problems occur when temperatures change with time. Such problems are often encountered in engineering practice, and the engineer may be required to predict the temperature-time response of a system involved in a heat transfer process. If the system, or a component of the system, can be assumed to have a spatially uniform temperature, analysis involves a relatively simple application of the energy conservation principle, as will now be demonstrated.

### 1.5.1 The Lumped Thermal Capacity Model

If a system undergoing a transient thermal response to a heat transfer process has a nearly uniform temperature, we may ignore small differences of temperature within the system. Changes in internal energy of the system can then be specified in terms of changes of the assumed uniform (or average) temperature of the system. This approximation is called the **lumped thermal capacity** model.<sup>7</sup> The system might

<sup>7</sup> The term *capacitance* is also used, in analogy to an equivalent electrical circuit.

be a small solid component of high thermal conductivity that loses heat slowly to its surroundings via a large external thermal resistance. Since the thermal resistance to conduction in the solid is small compared to the external resistance, the assumption of a uniform temperature is justified. Alternatively, the system might be a well-stirred liquid in an insulated tank losing heat to its surroundings, in which case it is the mixing of the liquid by the stirrer that ensures a nearly uniform temperature. In either case, once we have assumed uniformity of temperature, we have no further need for details of the heat transfer within the system—that is, of the conduction in the solid component or the convection in the stirred liquid. Instead, the heat transfer process of concern is the interaction of the system with the surroundings, which might be by conduction, radiation, or convection.

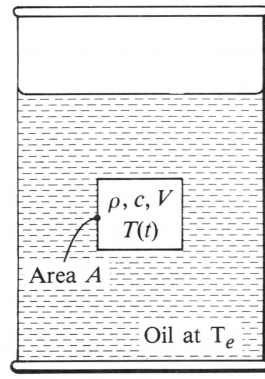
### ***Governing Equation and Initial Condition***

For purposes of analysis, consider a metal forging removed from a furnace at temperature  $T_0$  and suddenly immersed in an oil bath at temperature  $T_e$ , as shown in Fig. 1.16. The forging is a closed system, so the energy conservation principle in the form of Eq. (1.2) applies. Heat is transferred out of the system by convection. Using Eq. (1.24) the rate of heat transfer is  $\bar{h}_c A(T - T_e)$ , where  $\bar{h}_c$  is the heat transfer coefficient averaged over the forging surface area  $A$ , and  $T$  is the forging temperature. There is no heat generated within the forging, so that  $\dot{Q}_v = 0$ . Substituting in Eq. (1.2):

$$\begin{aligned} \rho V c \frac{dT}{dt} &= -\bar{h}_c A(T - T_e) \\ \frac{dT}{dt} &= -\frac{\bar{h}_c A}{\rho V c}(T - T_e) \end{aligned} \quad (1.36)$$

which is a first-order ordinary differential equation for the forging temperature,  $T$ , as a function of time,  $t$ . One initial condition is required:

$$t = 0: \quad T = T_0 \quad (1.37)$$



**Figure 1.16** A forging immersed in an oil bath for quenching.

### *Solution for the Temperature Response*

A simple analytical solution can be obtained provided we assume that the bath is large, so  $T_e$  is independent of time, and that  $\bar{h}_c A / \rho V c$  is approximated by a constant value independent of temperature. The variables in Eq. (1.36) can then be separated:

$$\frac{dT}{T - T_e} = -\frac{\bar{h}_c A}{\rho V c} dt$$

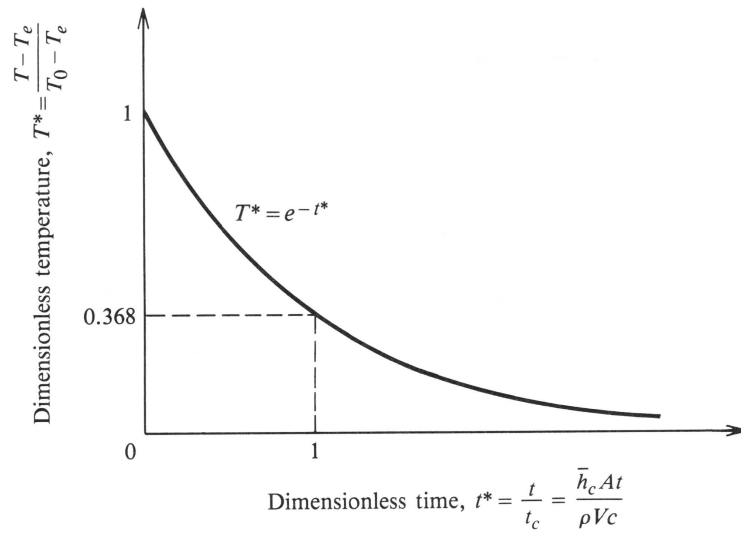
Writing  $dT = d(T - T_e)$ , since  $T_e$  is constant, and integrating with  $T = T_0$  at  $t = 0$ ,

$$\begin{aligned} \int_{T_0}^T \frac{d(T - T_e)}{T - T_e} &= -\frac{\bar{h}_c A}{\rho V c} \int_0^t dt \\ \ln \frac{T - T_e}{T_0 - T_e} &= -\frac{\bar{h}_c A}{\rho V c} t \\ \frac{T - T_e}{T_0 - T_e} &= e^{-(\bar{h}_c A / \rho V c)t} = e^{-t/t_c} \end{aligned} \quad (1.38)$$

where  $t_c = \rho V c / \bar{h}_c A$  [s] is called the **time constant** of the process. When  $t = t_c$ , the temperature difference  $(T - T_e)$  has dropped to be 36.8% of the initial difference  $(T_0 - T_e)$ . Our result, Eq. (1.38), is a relation between two dimensionless parameters: a dimensionless temperature,  $T^* = (T - T_e) / (T_0 - T_e)$ , which varies from 1 to 0; and a dimensionless time,  $t^* = t / t_c = \bar{h}_c A t / \rho V c$ , which varies from 0 to  $\infty$ . Equation (1.38) can be written simply as

$$T^* = e^{-t^*} \quad (1.39)$$

and a graph of  $T^*$  versus  $t^*$  is a single curve, as illustrated in Fig. 1.17.



**Figure 1.17** Lumped thermal capacity capacity temperature response in terms of dimensionless variables  $T^*$  and  $t^*$ .

Methods introduced in Chapter 2 can be used to deduce directly from Eqs. (1.36) and (1.37) that  $T^*$  must be a function of  $t^*$  alone [i.e.,  $T^* = f(t^*)$ ] without solving the equation. Of course, the solution also gives us the form of the function. Thus, the various parameters,  $\bar{h}_c$ ,  $c$ ,  $\rho$ , and so on, only affect the temperature response in the combination  $t^*$ , and not independently. If both  $\bar{h}_c$  and  $c$  are doubled, the temperature at time  $t$  is unchanged. This dimensionless parameter  $t^*$  is a dimensionless group in the same sense as the Reynolds number, but it does not have a commonly used name.

### *Validity of the Model*

We would expect our assumption of negligible temperature gradients within the system to be valid when the internal resistance to heat transfer is small compared with the external resistance. If  $L$  is some appropriate characteristic length of a solid body, for example,  $V/A$  (which for a plate is half its thickness), then

$$\frac{\text{Internal conduction resistance}}{\text{External convection resistance}} \simeq \frac{L/k_s A}{1/\bar{h}_c A} = \frac{\bar{h}_c L}{k_s} \simeq \frac{\bar{h}_c V}{k_s A} \quad (1.40)$$

where  $k_s$  is the thermal conductivity of the solid material. The quantity  $\bar{h}_c L/k_s$  [ $\text{W/m}^2 \text{K}$ ][m]/[ $\text{W/m K}$ ] is a dimensionless group called a **Biot number**,  $\text{Bi}$ .<sup>8</sup> More exact analyses of transient thermal response of solids indicate that, for bodies resembling a plate, cylinder, or sphere,  $\text{Bi}_{LTC} = \bar{h}_c V/k_s A < 0.1$  ensures that the temperature given by the lumped thermal capacity (LTC) model will not differ from the exact volume averaged value by more than 5%, and that our assumption of uniform temperature is adequate. Nonetheless, the choice of both the length scale  $L$  and the threshold (e.g.,  $\text{Bi}_{LTC} < 0.1$ ) used to determine the validity of the lumped thermal capacity model should be done carefully if accurate calculations are critical (see Chapter 3). If the heat transfer is by radiation, the convective heat transfer coefficient in Eq. (1.40) can be replaced by the approximate radiation heat transfer coefficient  $h_r$  defined in Eq. (1.19).

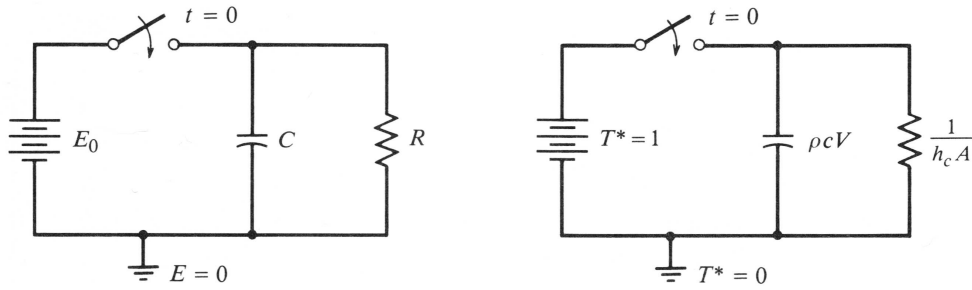
In the case of the well-stirred liquid in an insulated tank, it will be necessary to evaluate the ratio

$$\frac{\text{Internal convection resistance}}{\text{External resistance}} \simeq \frac{1/h_{c,i} A}{1/UA} = \frac{U}{h_{c,i}} \quad (1.41)$$

where  $U$  is the overall heat transfer coefficient, for heat transfer from the inner surface of the tank, across the tank wall and insulation, and into the surroundings. If this ratio is small relative to unity, the assumption of a uniform temperature in the liquid is justified.

The approximation or model used in the preceding analysis is called a lumped thermal capacity approximation since the thermal capacity is associated with a single temperature. There is an electrical analogy to the lumped thermal capacity model, owing to the mathematical equivalence of Eq. (1.36) to the equation governing the voltage in the simple resistance-capacitance electrical circuit shown

<sup>8</sup> To avoid confusion with the Biot number used in Chapter 3, we will denote the Biot number based on  $L = V/A$  as  $\text{Bi}_{LTC}$  for use with the lumped thermal capacity model.



**Figure 1.18** Equivalent electrical and thermal circuits for the lumped thermal capacity model of temperature response.

in Fig. 1.18,

$$\frac{dE}{dt} = -\frac{E}{RC} \quad (1.42)$$

with the initial condition  $E = E_0$  at  $t = 0$  if the capacitor is initially charged to a voltage  $E_0$ . The solution is identical in form to Eq. (1.38),

$$\frac{E}{E_0} = e^{-t/RC}$$

and the time constant is  $RC$ , the product of the resistance and capacitance [or  $C/(1/R)$ , the ratio of capacitance to conductance, to be exactly analogous to Eq. (1.38)].

### EXAMPLE 1.6 Quenching of a Steel Plate

A steel plate 1 cm thick is taken from a furnace at 600°C and quenched in a bath of oil at 30°C. If the heat transfer coefficient is estimated to be 400 W/m<sup>2</sup> K, how long will it take for the plate to cool to 100°C? Take  $k$ ,  $\rho$ , and  $c$  for the steel as 50 W/m K, 7800 kg/m<sup>3</sup> and 450 J/kg K, respectively.

#### Solution

**Given:** Steel plate quenched in an oil bath.

**Required:** Time to cool from 600°C to 100°C.

**Assumptions:** Lumped thermal capacity model valid.

First the Biot number will be checked to see if the lumped thermal capacity approximation is valid. For a plate of width  $W$ , height  $H$ , and thickness  $L$ ,

$$\frac{V}{A} \simeq \frac{WHL}{2WH} = \frac{L}{2}$$

where the surface area of the edges has been neglected.

$$\begin{aligned}
 \text{Bi}_{LTC} &= \frac{\bar{h}_c(L/2)}{k_s} \\
 &= \frac{(400 \text{ W/m}^2 \text{ K})(0.005 \text{ m})}{50 \text{ W/m K}} \\
 &= 0.04 < 0.1
 \end{aligned}$$

so the lumped thermal capacity model is applicable. The time constant  $t_c$  is

$$t_c = \frac{\rho V c}{\bar{h}_c A} = \frac{\rho(L/2)c}{\bar{h}_c} = \frac{(7800 \text{ kg/m}^3)(0.005 \text{ m})(450 \text{ J/kg K})}{(400 \text{ W/m}^2 \text{ K})} = 43.9 \text{ s}$$

Substituting  $T_e = 30^\circ\text{C}$ ,  $T_0 = 600^\circ\text{C}$ ,  $T = 100^\circ\text{C}$  in Eq. (1.38) gives

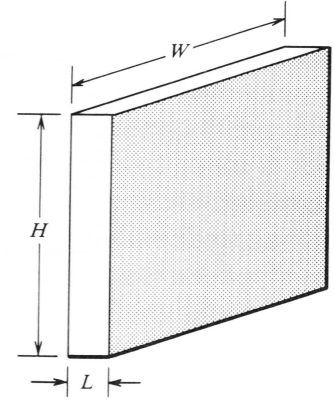
$$\frac{100 - 30}{600 - 30} = e^{-t/43.9}$$

Solving,

$$t = 92 \text{ s}$$

### Comments

The use of a constant value of  $\bar{h}_c$  may be inappropriate for heat transfer by natural convection or radiation.

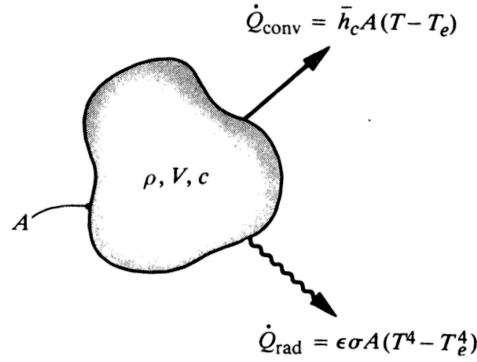


## 1.5.2 Combined Convection and Radiation

The analysis of Section 1.5.1 assumes that the heat transfer coefficient was constant during the cooling period. This assumption is adequate for forced convection but is less appropriate for natural convection, and when thermal radiation is significant. Equation (1.23) shows that the natural convection heat transfer coefficient  $\bar{h}_c$  is proportional to  $\Delta T^{1/4}$  for laminar flow and to  $\Delta T^{1/3}$  for turbulent flow. The temperature difference  $\Delta T = T - T_e$  decreases as the body cools, as does  $\bar{h}_c$ . Radiation heat transfer is proportional to  $(T^4 - T_e^4)$  and hence cannot be represented exactly by Newton's law of cooling. We now extend our lumped thermal capacity analysis to allow both for a variable convective heat transfer coefficient and for situations where both convection and radiation are important.

### Governing Equation and Initial Condition

Figure 1.19 shows a body that loses heat by both convection and radiation. For a small gray body in large, nearly black surroundings also at temperature  $T_e$ , the radiation heat transfer is obtained from Eq. (1.18) as  $\dot{Q} = \varepsilon A \sigma (T^4 - T_e^4)$ . As in



**Figure 1.19** Schematic of a body losing heat by convection and radiation for a lumped thermal capacity model

Section 1.5.1, the energy conservation principle, Eq. (1.2), becomes

$$\rho V c \frac{dT}{dt} = -\bar{h}_c A (T - T_e) - \epsilon A \sigma (T^4 - T_e^4)$$

or

$$\frac{dT}{dt} = -\frac{\bar{h}_c A}{\rho V c} (T - T_e) - \frac{\epsilon A \sigma}{\rho V c} (T^4 - T_e^4) \quad (1.43)$$

The initial condition is again

$$t = 0: \quad T = T_0 \quad (1.44)$$

This first-order ordinary differential equation has no convenient analytical solution even when the convective heat transfer coefficient  $\bar{h}_c$  is constant as in forced convection. However, Eq. (1.43) can be solved easily using a numerical integration procedure. For this purpose, it can be rearranged as

$$\frac{dT}{dt} + \frac{hA}{\rho V c} (T - T_e) = 0 \quad (1.45)$$

$$h = \bar{h}_c + h_r = B(T - T_e)^n + \sigma \epsilon (T^2 + T_e^2)(T + T_e) \quad (1.46)$$

where  $(T^4 - T_e^4)$  has been factored, as was done in deriving Eq. (1.19). For forced convection,  $n = 0$ ,  $B = \bar{h}_c$ ; for laminar natural convection  $n = 1/4$  and  $B$  is a constant [for example, for a plate of height  $L$ , Eq. (1.23a) gives  $B = (4/3)(1.07)/L^{1/4}$ ]. Equation (1.46) defines a total heat transfer coefficient that accounts for both convection and radiation and changes continuously as the body cools. To put Eq. (1.45) in dimensionless form, we use the dimensionless variables introduced in Section 1.5.1:

$$T^* = \frac{T - T_e}{T_0 - T_e}, \quad t^* = \frac{t}{t_c} \quad (1.47a,b)$$

The definition of the time constant  $t_c$  poses a problem since  $h$  is not a constant as before. We choose to define  $t_c$  in terms of the value of  $h$  at time  $t = 0$ , when the body temperature is  $T_0$ ,

$$t_c = \frac{\rho V c}{h_0 A} = \frac{\rho V c}{[B(T_0 - T_e)^n + \sigma \epsilon (T_0^2 + T_e^2)(T_0 + T_e)]A} \quad (1.48)$$

Equation (1.45) then becomes

$$\frac{dT^*}{dt^*} + \frac{h(T^*)}{h_0} T^* = 0 \quad (1.49)$$

with the initial condition

$$t^* = 0: \quad T^* = 1 \quad (1.50)$$

### Computer Program LUMP

Numerical integration is appropriate for this problem. The computer program LUMP has been prepared accordingly. LUMP solves Eq. (1.49), that is, it obtains the temperature response of a body that loses heat by convection and/or radiation, based on the lumped thermal capacity model. The required input constant  $B$  is defined in Eq. (1.46). Any consistent system of units can be used. The output can be obtained either as graph or as numerical data.

---

### EXAMPLE 1.7 Quenching of an Alloy Sphere

A materials processing experiment under microgravity conditions on the space station requires quenching in a forced flow of an inert gas. A 1 cm-diameter metal alloy sphere is removed from a furnace at 800°C and is to be cooled to 500°C by a flow of nitrogen gas at 25°C. Determine the effect of the convective heat transfer coefficient on cooling time for  $10 < \bar{h}_c < 100 \text{ W/m}^2 \text{ K}$ . Properties of the alloy include:  $\rho = 14,000 \text{ kg/m}^3$ ;  $c = 140 \text{ J/kg K}$ ;  $\varepsilon = 0.1$ . The surrounds can be taken as nearly black at 25°C.

#### Solution

**Given:** A metal alloy sphere to be quenched.

**Required:** Effect of convective heat transfer coefficient on cooling time.

**Assumptions:** 1. Lumped thermal capacity model valid.  
2. Constant convective heat transfer coefficient.

The computer code LUMP can be used to solve this problem.

The required inputs are:

$$T_0 \text{ and } T_e = 1073, 298$$

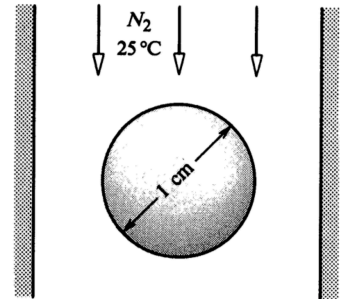
$$B = \bar{h}_c = 10$$

$$n = 0$$

$$\sigma = 5.67 \times 10^{-8}$$

$$\varepsilon = 0.1$$

$$\text{Final value of } t^*: \text{ try } t^* = 1$$



The required dimensionless temperature is

$$T^* = \frac{T - T_e}{T_0 - T_e} = \frac{773 - 298}{1073 - 298} = 0.613$$

and the code is used to obtain the corresponding dimensionless time  $t^*$ . For a sphere  $V/A = (\pi D^3/6)/(\pi D^2) = D/6$ , so that the time constant is

$$t_c = \frac{\rho V c}{h_0 A} = \frac{\rho (D/6) c}{h_0} = \frac{(14,000)(0.01/6)(140)}{\bar{h}_c + (5.67 \times 10^{-8})(0.1)(1073^2 + 298^2)(1073 + 298)}$$

$$= \frac{3267}{\bar{h}_c + 9.64} \text{ s}$$

The actual time is  $t = t^* t_c$ . Results obtained using LUMP are tabulated below.

$\bar{h}_c$ W/m <sup>2</sup> K	$t^*$	$t_c$ s	$t$ s
10	0.59	166	98
20	0.54	110	61
30	0.53	82	43
50	0.52	55	29
100	0.51	30	15

### Comments

1. Only two significant figures have been given since high accuracy is not warranted for the problem.
2. The heat transfer coefficient does not have a strong effect on  $t^*$ . Why?
3. For the lumped thermal capacity model to be valid, the Biot number,  $Bi_{LTC}$ , should be less than 0.1. The worst case is with  $\bar{h}_c = 100 \text{ W/m}^2 \text{ K}$  at time  $t = 0$ , giving  $h_0 = 109.6 \text{ W/m}^2 \text{ K}$  and  $0.1 > (109.6)(0.01/6)/k_s$ , that is,  $k_s > 1.8 \text{ W/m K}$ , which certainly will be true for a metal alloy.

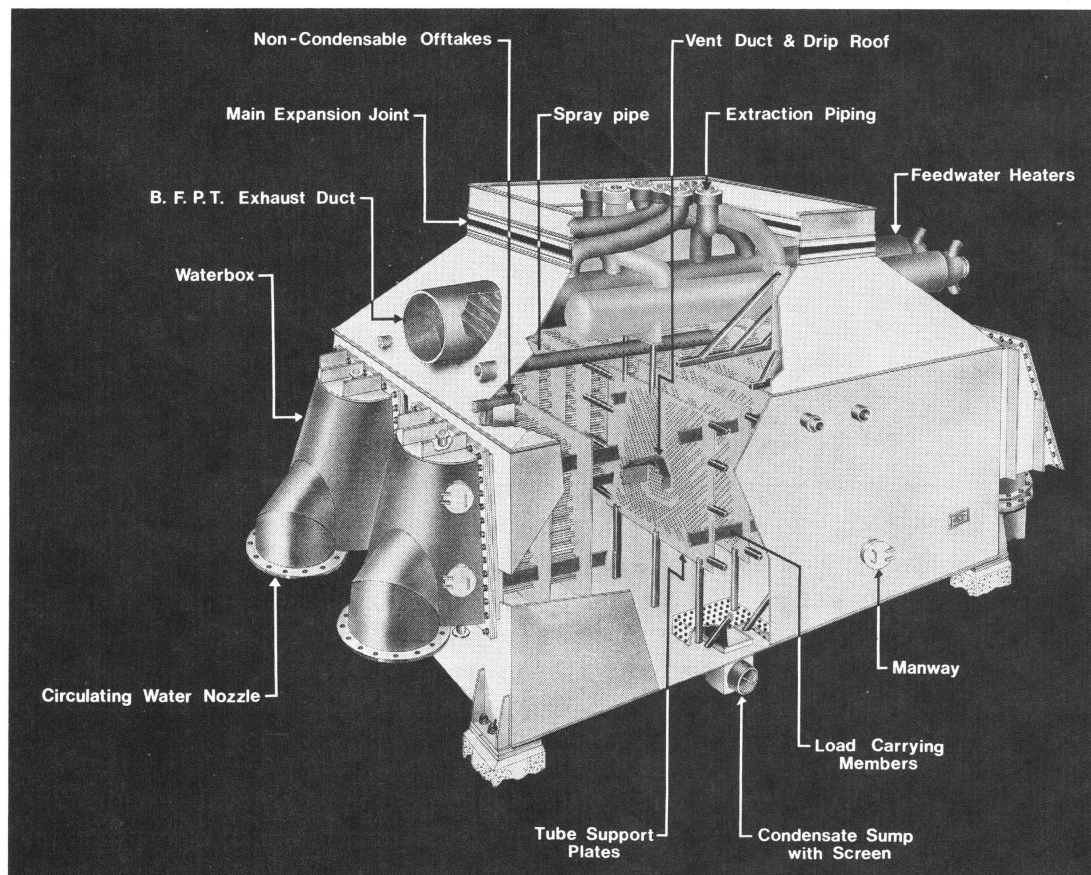
## 1.6 HEAT EXCHANGERS

In Section 1.5, we considered problems in which the temperature of a system changed with time as a result of heat transfer between the system and its surroundings. We now consider problems in which the temperature of a fluid changes as it flows through a passage as a result of heat transfer between the passage walls and the fluid. These problems are encountered in the analysis of heat exchanger performance. A *heat exchanger* is a device that facilitates transfer of heat from one fluid stream to another. Power production, refrigeration, heating and air conditioning, food processing, chemical processing, oil refining, and the operation of almost all vehicles depends on heat exchangers of various types. The analysis and design of heat exchangers is the subject of Chapter 8. The analysis of a very simple heat exchanger configuration is presented here to introduce some of the basic concepts underlying heat exchanger analysis and associated terminology. These concepts will prove useful in the development and application of heat transfer theory in chapters preceding Chapter 8—particularly in Chapters 4 and 5, which deal with convection.

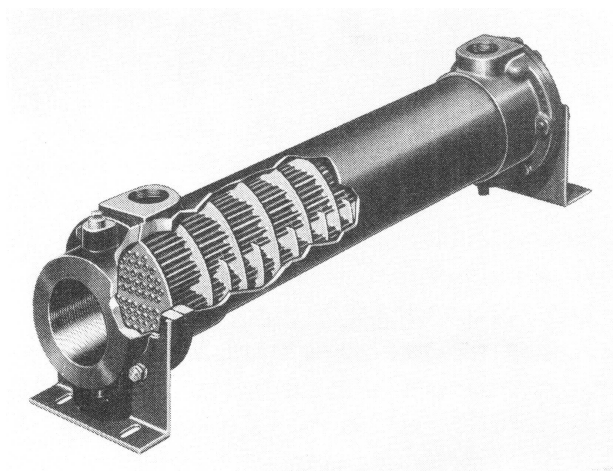
### 1.6.1 Single- and Two-Stream Exchangers

One important classification of heat exchangers is into **single-stream exchangers** and **two-stream exchangers**. A single-stream exchanger is one in which the temperature of only one stream changes in the exchanger; examples include many types of evaporators and condensers found in power plants and refrigeration systems. A power plant condenser is shown in Fig. 1.20. A two-stream exchanger is one in which the temperatures of both streams change in the exchanger; examples include radiators and intercoolers for automobile engines, and oil coolers for aircraft engines. Figure 1.21 shows an oil cooler, which has a **counterflow** configuration; that is, the streams flow in opposite directions in the exchanger.

In the analysis of heat exchangers, a useful first step is to draw a sketch of the expected fluid temperature variations along the exchanger. Figure 1.22*a* is such a sketch for the power plant condenser. The hot stream is steam returning from the turbines, which condenses at a constant temperature  $T_H$ . This is the saturation temperature corresponding to the pressure maintained in the condenser shell. The cold stream is cold water from a river, ocean, or cooling tower, and its temperature  $T_C$



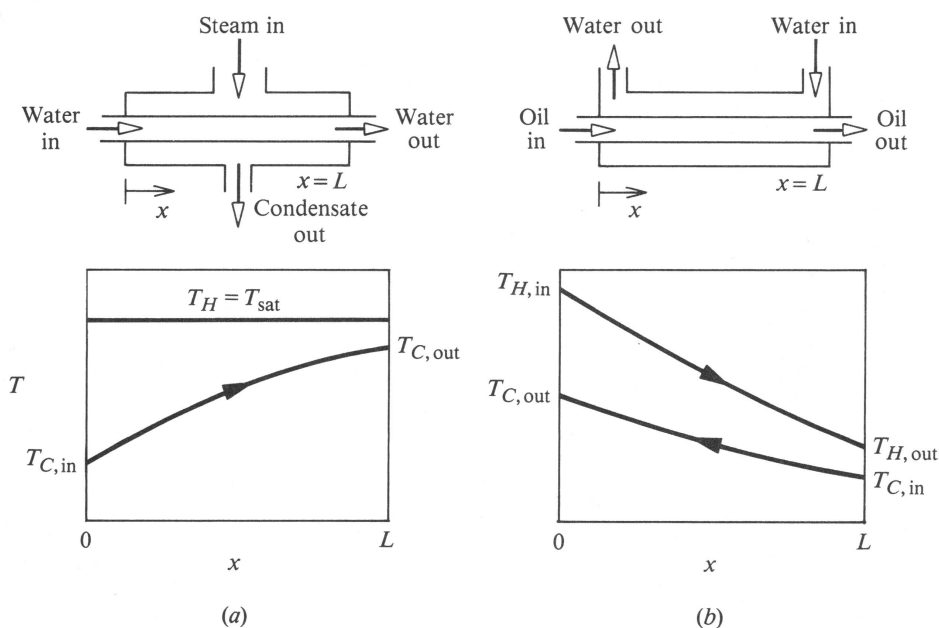
**Figure 1.20** A power plant condenser. (Courtesy Senior Engineering Co. [formerly Southwestern Engineering], Los Angeles, California.)



**Figure 1.21** A small single-pass shell-and-tube two-stream heat exchanger, typically used for cooling oil or water. (Photograph courtesy of the Young Radiator Company, Racine, Wis.)

increases as it flows through the exchanger. Figure 1.22*b* shows the sketch for the oil cooler. The hot stream is oil from the engine, and the cold stream is coolant water. Notice that in this counterflow configuration, the cold stream can leave the exchanger at a higher temperature than the leaving hot stream!

A point that might confuse the beginning student is that there are actually two streams in many single-stream exchangers. The definition simply requires that the temperature of only one stream changes in a single-stream exchanger. It is this feature that makes the analysis of single-stream exchangers particularly simple, as will now be demonstrated. In Section 1.5, the system analysis was based on the

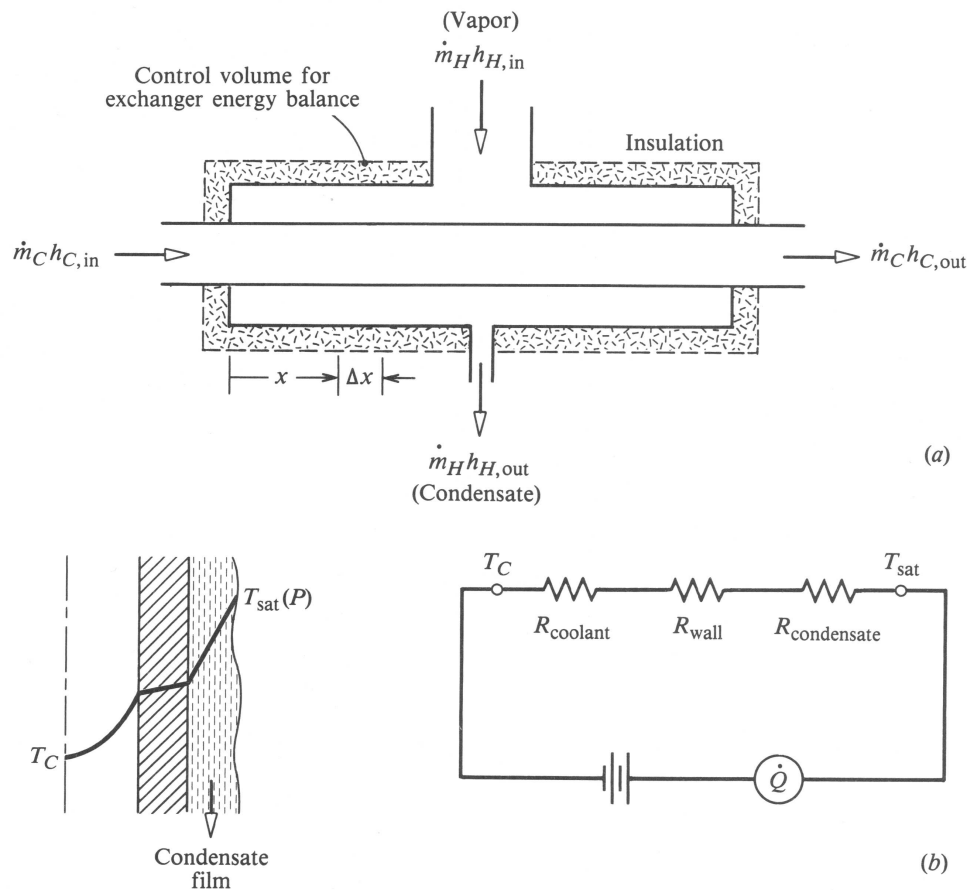


**Figure 1.22** Temperature variations along heat exchangers. (a) A power plant condenser. (b) A counterflow oil cooler.

energy conservation principle in the form of the first law of thermodynamics applied to a *closed system*. In contrast, the system analysis that follows is based on the first law applied to an *open system*.

### 1.6.2 Analysis of a Condenser

Figure 1.23a shows a simple single-tube condenser. Pure saturated vapor enters the shell at the top and condenses on a single horizontal tube. The condensate forms a thin film on the outside of the tube, drops off the bottom, and leaves the shell through a drain. The vapor condenses at the saturation temperature corresponding to the pressure in the shell. Hence, the condensate film surface temperature is  $T_{\text{sat}}(P)$ . Figure 1.23b shows the temperature variation across the tube wall and the corresponding thermal circuit. The enthalpy of condensation is transferred by conduction across the thin condensate film, by conduction across the tube wall; and by convection into the coolant. As a result, the coolant temperature rises as it gains energy flowing along the tube. The vapor flow rate is denoted  $\dot{m}_H$  [kg/s] and the coolant flow rate  $\dot{m}_C$  (the *hot* and *cold* streams, respectively).



**Figure 1.23** (a) Schematic of a single-tube condenser. (b) The temperature variation across the tube wall and the thermal circuit for heat transfer across the tube wall.

### The Exchanger Energy Balance

An energy balance on the exchanger as a whole is formulated by writing down the steady-flow energy equation for a control volume enclosing the exchanger (the dashed line in Fig. 1.23a). If the exchanger is well insulated, there is no heat loss to the surroundings, and Eq. (1.4) requires that the enthalpy inflow equal the enthalpy outflow:

$$\dot{m}_H h_{H,\text{in}} + \dot{m}_C h_{C,\text{in}} = \dot{m}_H h_{H,\text{out}} + \dot{m}_C h_{C,\text{out}}$$

where  $h$  is specific enthalpy [J/kg] and subscripts “in” and “out” denote inlet and outlet values, respectively. Rearranging gives

$$\dot{m}_C (h_{C,\text{out}} - h_{C,\text{in}}) = \dot{m}_H (h_{H,\text{in}} - h_{H,\text{out}}) \quad (1.51)$$

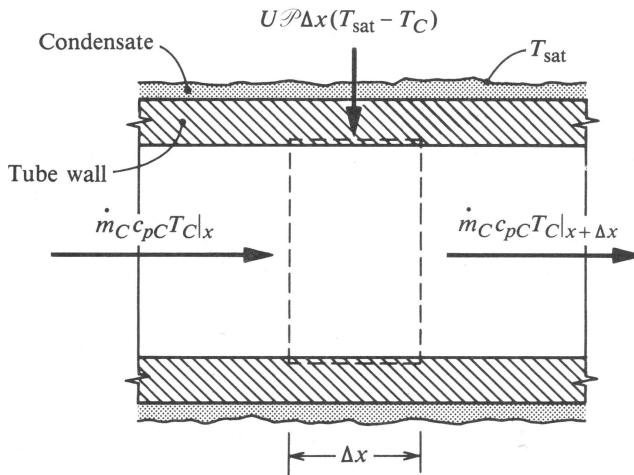
If we assume a constant specific heat for the coolant and that the condensate leaves at the saturation temperature, Eq. (1.51) becomes

$$\dot{m}_C c_{pC} (T_{C,\text{out}} - T_{C,\text{in}}) = \dot{m}_H h_{\text{fg}} \quad (1.52)$$

where  $h_{\text{fg}}$  is the enthalpy of vaporization for the vapor. When the coolant flow rate  $\dot{m}_C$  and inlet temperature  $T_{C,\text{in}}$  are known, Eq. (1.52) relates the coolant outlet temperature  $T_{C,\text{out}}$  to the amount of vapor condensed  $\dot{m}_H$ .

### Governing Equation and Boundary Condition

To determine the variation of coolant temperature along the exchanger, we make an energy balance on a differential element of the exchanger  $\Delta x$  long and so derive a differential equation with  $x$  as the independent variable and  $T_C$  as the dependent variable. When the steady-flow energy equation, Eq. (1.4), is applied to the control volume of length  $\Delta x$ , shown in Fig. 1.24 as a dotted line, the contribution to  $\dot{Q}$  due to  $x$ -direction conduction in the coolant is small and can be neglected. Thus, the



**Figure 1.24** An elemental control volume  $\Delta x$  long for application of the steady-flow energy equation to a condenser coolant stream.

coolant flow rate times its enthalpy increase must equal the heat transfer across the tube wall:

$$\dot{m}_{CC} c_{pC} (T_C|_{x+\Delta x} - T_C|_x) = U \mathcal{P} \Delta x (T_{\text{sat}} - T_C)$$

where  $U [\text{W/m}^2 \text{ K}]$  is the overall heat transfer coefficient for heat transfer from the vapor to the coolant, and  $\mathcal{P} [\text{m}]$  is the perimeter of the tube wall. Thus,  $\mathcal{P} \Delta x$  is the surface area of the tube element with length  $\Delta x$ . For a circular tube  $\mathcal{P} = \pi D$  where  $D$  is the pipe diameter. Dividing by  $\Delta x$ ,

$$\dot{m}_{CC} c_{pC} \left( \frac{T_C|_{x+\Delta x} - T_C|_x}{\Delta x} \right) = U \mathcal{P} (T_{\text{sat}} - T_C)$$

and letting  $\Delta x \rightarrow 0$ , gives

$$\dot{m}_{CC} c_{pC} \frac{dT_C}{dx} = U \mathcal{P} (T_{\text{sat}} - T_C)$$

Rearranging,

$$\frac{dT_C}{dx} - \frac{U \mathcal{P}}{\dot{m}_{CC} c_{pC}} (T_{\text{sat}} - T_C) = 0 \quad (1.53)$$

Equation (1.53) is a first-order ordinary differential equation for  $T_C(x)$ ; it requires one boundary condition, which is

$$x = 0; \quad T_C = T_{C,\text{in}} \quad (1.54)$$

### Temperature Variation

To integrate Eq. (1.53), let  $\theta = T_{\text{sat}} - T_C$ ; then  $dT_C/dx = -d\theta/dx$ , and the equation becomes

$$\frac{d\theta}{dx} + \frac{U \mathcal{P}}{\dot{m}_{CC} c_{pC}} \theta = 0$$

If  $U$  is assumed constant along the exchanger, the solution is

$$\theta = A e^{-(U \mathcal{P} / \dot{m}_{CC} c_{pC}) x}$$

where  $A$  is the integration constant. Substituting for  $\theta$  and using the boundary condition, Eq. (1.54) gives the integration constant:

$$T_{\text{sat}} - T_{C,\text{in}} = A e^0 = A$$

Thus, the solution of Eq. (1.53) is

$$T_{\text{sat}} - T_C = (T_{\text{sat}} - T_{C,\text{in}}) e^{-(U \mathcal{P} / \dot{m}_{CC} c_{pC}) x} \quad (1.55)$$

which is the desired relation  $T_C(x)$ , showing an exponential variation along the exchanger. Of particular interest is the coolant outlet temperature  $T_{C,\text{out}}$  which is obtained by letting  $x = L$ , the length of the exchanger, in Eq. (1.55):

$$T_{\text{sat}} - T_{C,\text{out}} = (T_{\text{sat}} - T_{C,\text{in}}) e^{-(U \mathcal{P} L / \dot{m}_{CC} c_{pC})} \quad (1.56)$$

### Exchanger Performance Parameters

The product of perimeter and length  $\mathcal{P}L$  is the area of the heat transfer surface. The exponent in Eq. (1.56) is, of course, dimensionless,

$$\left[ \frac{U \mathcal{P}L}{\dot{m}_{CC} c_{pC}} \right] = \frac{[\text{W/m}^2 \text{ K}][\text{m}][\text{m}]}{[\text{kg/s}][\text{J/kg K}]} = \left[ \frac{\text{W s}}{\text{J}} \right] = 1$$

since a watt is a joule per second. This dimensionless group is called the **number of transfer units**, with abbreviation NTU and symbol  $N_{tu}$ .<sup>9</sup> For a given  $\dot{m}_{CC} c_{pC}$ , the larger  $U$ ,  $\mathcal{P}$ , or  $L$ , the greater the NTU of the exchanger. Thus, the NTU can be viewed as a measure of the heat transfer “size” of the exchanger. Equation (1.56) can then be rearranged as

$$\frac{T_{\text{sat}} - T_{C,\text{out}}}{T_{\text{sat}} - T_{C,\text{in}}} = e^{-N_{tu}} \quad (1.57)$$

Thus, if  $T_{\text{sat}}$ ,  $T_{C,\text{in}}$ , and the NTU of the exchanger are known,  $T_{C,\text{out}}$  can be calculated. But we find it convenient to rearrange Eq. (1.57) by subtracting each side from unity to obtain

$$1 - \frac{T_{\text{sat}} - T_{C,\text{out}}}{T_{\text{sat}} - T_{C,\text{in}}} = 1 - e^{-N_{tu}}$$

or

$$\frac{T_{C,\text{out}} - T_{C,\text{in}}}{T_{\text{sat}} - T_{C,\text{in}}} = 1 - e^{-N_{tu}} \quad (1.58)$$

Now, even if the exchanger were infinitely long, the maximum outlet temperature of the coolant would be  $T_{\text{sat}}$  (see Fig. 1.22a). Thus, the left-hand side of Eq. (1.58) is the ratio of the actual temperature rise of the coolant ( $T_{C,\text{out}} - T_{C,\text{in}}$ ) divided by the maximum possible rise for an infinitely long exchanger ( $T_{\text{sat}} - T_{C,\text{in}}$ ) and can be viewed as the **effectiveness** of the exchanger, for which we use the symbols  $\varepsilon$ . Our result is therefore

$$\varepsilon = 1 - e^{-N_{tu}} \quad (1.59)$$

Equation (1.59) indicates that the larger the number of transfer units of the exchanger, the higher its effectiveness. Although a high effectiveness is desirable, as the length of an exchanger increases, so does the cost of materials for its construction and the pumping power required by the coolant flow. Thus, the goal of the design engineer is to maximize the effectiveness subject to the constraints of construction (capital) costs and power (operating) costs. In practice, values of  $\varepsilon$  between 0.6 and 0.9 are typical.

<sup>9</sup> NTU is also widely used as the symbol for number of transfer units.

**EXAMPLE 1.8** Performance of a Steam Condenser

A steam condenser is 4 m long and contains 2000, 5/8 inch nominal-size, 18 gage brass tubes (1.59 cm O.D., 1.25 mm wall thickness). In a test 120 kg/s of coolant water at 300 K is supplied to the condenser, and when the steam pressure in the shell is 10,450 Pa, condensate is produced at a rate of 3.02 kg/s. Determine the effectiveness of the exchanger and the overall heat transfer coefficient. Take the specific heat of the water to be 4174 J/kg K.

**Solution**

**Given:** A shell-and-tube steam condenser.

**Required:** The effectiveness,  $\epsilon$ , and overall heat transfer coefficient,  $U$ .

**Assumptions:**  $U$  is constant along the exchanger so that Eq. (1.59) applies.

The hot-stream temperature  $T_H$  is the saturation temperature corresponding to the given steam pressure of 10,540 Pa; from steam tables (see Table A.12a in Appendix A of this text)  $T_{\text{sat}} = 320.0$  K. We first find the coolant water outlet temperature from the exchanger energy balance Eq. (1.52):

$$\dot{m}_C c_{pC} (T_{C,\text{out}} - T_{C,\text{in}}) = \dot{m}_H h_{\text{fg}}$$

From steam tables, the enthalpy of vaporization at  $T_{\text{sat}} = 320$  K is  $h_{\text{fg}} = 2.389 \times 10^6$  J/kg.

$$(120 \text{ kg/s})(4174 \text{ J/kg K})(T_{C,\text{out}} - 300 \text{ K}) = (3.02 \text{ kg/s})(2.389 \times 10^6 \text{ J/kg})$$

Solving gives  $T_{C,\text{out}} = 314.4$  K.

The effectiveness,  $\epsilon$ , is then obtained from Eq. (1.58) as

$$\epsilon = \frac{T_{C,\text{out}} - T_{C,\text{in}}}{T_{\text{sat}} - T_{C,\text{in}}} = \frac{314.4 - 300}{320 - 300} = 0.720$$

and the number of transfer units, from Eq. (1.59), is

$$N_{\text{tu}} = \ln \frac{1}{1 - \epsilon} = \ln \frac{1}{1 - 0.720} = 1.27 = \frac{U \mathcal{P}L}{\dot{m}_C c_{pC}}$$

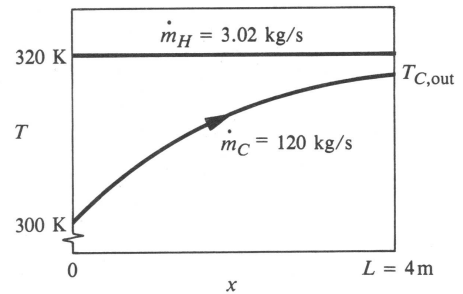
Solving for the  $U \mathcal{P}L$  product,

$$U \mathcal{P}L = 1.27 \dot{m}_C c_{pC} = (1.27)(120 \text{ kg/s})(4174 \text{ J/kg K}) = 6.36 \times 10^5 \text{ W/K}$$

If we choose to base the overall heat transfer coefficient on the outside area of the tubes, then, for  $N$  tubes, the heat transfer area  $\mathcal{P}L$  is

$$\mathcal{P}L = N\pi DL = (2000)(\pi)(1.59 \times 10^{-2} \text{ m})(4 \text{ m}) = 400 \text{ m}^2$$

Hence,  $U = U \mathcal{P}L / \mathcal{P}L = 6.36 \times 10^5 / 400 = 1590 \text{ W/m}^2 \text{ K}$

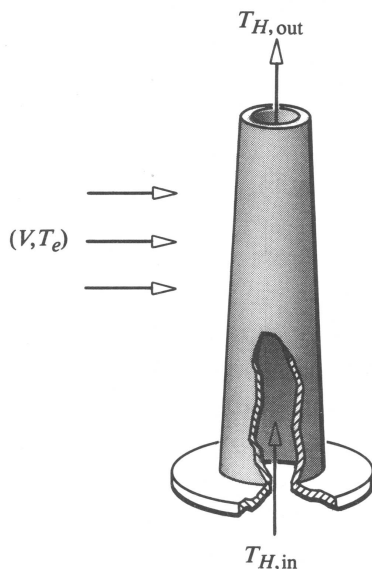


**Comments**

We could have performed these calculations by considering a single tube of the tube bundle, for which the coolant flow is  $(120/2000)$  kg/s and the heat transfer area is simply  $\pi DL$ . But common practice is always to consider the exchanger as a whole, as we have done here.

**1.6.3 Other Single-Stream Exchangers**

Simple evaporators and boilers are also single-stream exchangers, where the cold stream is an evaporating or boiling liquid and the hot stream supplies the enthalpy of vaporization. Such exchangers will be analyzed in Chapter 8. Heat transfer to a fluid stream may also be a concern in problems that do not involve heat exchangers. The exhaust gas stack cooled by a crosswind, shown in Fig. 1.25, can also be viewed as a single-stream heat exchanger, since only the exhaust gas temperature changes with location up the stack. Thus, the analysis of Section 1.6.2, properly interpreted, applies (see Exercise 1–52). Single-stream heat exchanger theory also will be used in Chapters 4 and 5 in the examination of convective heat transfer in internal flows.



**Figure 1.25** An exhaust gas stack cooled by a crosswind.

**1.7 DIMENSIONS AND UNITS**

*Dimensions* are physical properties that are measurable — for example, length, time, mass, and temperature. A system of *units* is used to give numerical values to dimensions. The system most widely used throughout the world in science and industry is the International System of units (SI), from the French name *Système International d'Unités*. This system was recommended at the General Conference on Weights and Measures of the International Academy of Sciences in 1960 and was adopted by the U.S. National Bureau of Standards in 1964.

In the United States, the transition from the older English system of units to the SI system has been slow and is not complete. The SI system is used in science education, in government contracts, by engineering professional societies, and by many industries. However, engineers in some more mature industries still prefer to use English units, and, of course, commerce and trade in the United States remains dominated by the English system. We buy pounds of vegetables, quarts of milk, drive miles to work, and say that it is a hot day when the temperature exceeds 80°F. (Wine is now sold in 750 ml bottles, though, which is a modest step forward!)

In this text, we will use the SI system, with which the student has become familiar from physics courses. For convenience, this system is summarized in the tables of Appendix B. Base and supplementary units, such as length, time, and plane angle, are given in Table B.1a; and derived units, such as force and energy, are given in Table B.1b. Recognized non-SI units (e.g., hour, bar) that are acceptable for use with the SI system are listed in Table B.1c. Multiples of SI units (e.g., kilo, micro) are defined in Table B.1d. Accordingly, the property data given in the tables of Appendix A are in SI units. The student should review this material and is urged to be careful when writing down units. For example, notice that the unit of temperature is a kelvin (not Kelvin) and has the symbol K (not °K). Likewise, the unit of power is the watt (not Watt). The symbol for a kilogram is kg (not KG). An issue that often confuses the student is the correct use of Celsius temperature. Celsius temperature is defined as  $(T - 273.15)$  where  $T$  is in kelvins. However, the unit “degree Celsius” is equal to the unit “kelvin” ( $1^{\circ}\text{C} = 1 \text{ K}$ ).

Notwithstanding the wide acceptance of the SI system of units, there remains a need to communicate with those engineers (or lawyers!) who are still using English units. Also, component dimensions, or data for physical properties, may be available only in English or cgs units. For example, most pipes and tubes used in the United States conform to standard sizes originally specified in English units. A 1 inch nominal-size tube has an outside diameter of 1 in. For convenience, selected dimensions of U.S. commercial standard pipes and tubes are given in SI units in Appendix A as Tables A.14a and A.14b, respectively. The engineer must be able to convert dimensions from one system of units to another. Table B.2 in Appendix B gives the conversion factors required for most heat transfer applications. The program UNITS is based on Table B.2 and contains all the conversion factors in the table. With the input of a quantity in one system of units, the output is the same quantity in the alternative units listed in Table B.2. It is recommended that the student or engineer perform all problem solving using the SI system so as to efficiently use the Appendix A property data and the computer software. If a problem is stated in English units, the data should be converted to SI units using UNITS; if a customer requires results in units other than SI, UNITS will give the required values.

## 1.8 CLOSURE

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Chapter 1 had two main objectives:

1. To introduce the three important modes of heat transfer, namely, conduction, radiation, and convection.
2. To demonstrate how the first law of thermodynamics is applied to an engineering system to obtain the consequences of a heat transfer process.

For each mode of heat transfer, some working equations were developed, which, though simple, allow heat transfer calculations to be made for a wide variety of problems. Equations (1.9), (1.18), and (1.20) are probably the most frequently used equations for thermal design. An electric circuit analogy was shown to be a useful aid for problem solving when more than one mode of heat transfer is involved. In applying the first law to engineering systems, both a closed system and an open system were considered. In the first case, the variation of temperature with time was determined for a solid of high conductivity or a well-stirred fluid. In the second case, the variation of fluid temperature with position along the heat exchanger tube was determined.

The student should be familiar with some of the Chapter 1 concepts from previous physics, thermodynamics, and fluid mechanics courses. A review of texts for such courses is appropriate at this time. Many new concepts were introduced, however, which will take a little time and effort to master. Fortunately, the mathematics in this chapter is simple, involving only algebra, calculus, and the simplest first-order differential equation, and should present no difficulties to the student. After successfully completing a selection of the following exercises, the student will be well equipped to tackle subsequent chapters.

A feature of this text is an emphasis on real engineering problems as examples and exercises. Thus, Chapter 1 has somewhat greater scope and detail than the introductory chapters found in most similar texts. With the additional material, more realistic problems can be treated, both in Chapter 1 and in subsequent chapters. In particular, conduction problems in Chapters 2 and 3 have more realistic convection and radiation boundary conditions. Convective heat transfer coefficients for flow over tube bundles in Chapter 4 are calculated in the appropriate context of a heat exchanger. Similarly, condensation heat transfer coefficients in Chapter 7 can be discussed in the context of condenser performance. Throughout the text are exercises that require application of the first law to engineering systems, for it is always the consequences of a heat transfer process that motivate the engineer's concern with the subject.

Two computer programs accompany Chapter 1. The program LUMP calculates temperature response using the lumped thermal capacity model of Section 1.5. When heat loss is by convection and radiation simultaneously, the problem does not have an analytical solution. However, a numerical solution is easily obtained; LUMP demonstrates the value of writing a computer program in such situations. It is important that engineering students be aware of the potential of scientific computing as an engineering tool and take the initiative to use computational methods when appropriate. The program UNITS is a simple units conversion tool that allows unit conversions to be made quickly and reliably.